FIE402: Workshop 3

Chapter 20: Financial options

Example

In mid-February 2016, European-style options on the S&P index (OEX) expiring in December 2017 were priced as follows:

December 2017 OEX Options		
Strike Price	Call Price	Put Price
830	87.76	?
850	75.93	101.88
870	?	112.31

Given an interest rate of 0.42 % (0.0042) for a December 2017 maturity (22 months in the future), use put-call parity (with dividends) to determine:

A. The price of a December 2017 OEX put option with a strike price of 830.

By the law of one price, we have that

$$S + P = PV(K) + C$$

where S is the stock price, P is the put price, K is the strike price of the option (the price you want to ensure that the stock will not drop below), and C is the call price.

In the case a stock pays dividends, however, the equation is as follows:

$$S - PV(Div) + P = PV(K) + C$$

We want to find the price of a put options (P) with a strike price of 830. By rearranging the put-call parity we get the following:

$$P = PV(K) + C - S + PV(Div)$$

With a strike price of 830 we know from the table that the price of a call option (C) is 87.76. However, in order to use the put-call parity with dividends, we also need to know the stock price minus the dividends. The first step is therefore to solve for $S - PV(Div)$.

Step 1: Solve for $S - PV(Div)$

Solving for $S - PV(Div)$ gives us:

$$S - PV(Div) + P = PV(K) + C$$

$$S - PV(Div) = PV(K) + C - P$$

Now, we can use the information from the 850 calls and puts to solve for $S - PV(Div)$.

$$S - PV(Div) = PV(K) + C - P$$

$$S - PV(Div) = \frac{850}{1.0042^{\frac{22}{12}}} + 75.93 - 101.88 = 817.54$$

Step 2: Solve for the 830 put

We use the put-call parity with dividends

$$S - PV(Div) + \boldsymbol{P} = PV(K) + C$$

and then we plug in the information we have, namely

$$\underbrace{817.54}_{S - PV(Div)} + \boldsymbol{P} = \underbrace{\frac{830}{1.0042^{\frac{22}{12}}}}_{PV(K)} + \underbrace{87.76}_{C}$$

Then we solve for P:

$$P = \frac{830}{1.0042^{\frac{22}{12}}} + 87.76 - 817.54 = 93.87$$

The price of a December 2017 OEX put option with a strike price of 830 is 93.87.

B. The price of a December 2017 OEX call option with a strike price of 870.

To calculate the 870 call we again use the put-call parity with dividends:

$$S - PV(Div) + P = PV(K) + \mathbf{C}$$

From question A we know that $S - PV(Div)$ is equal to 817.54. With a strike price of 870 the price of a put option (P) is 112.31. Now we can plug this information in the formula

$$\underbrace{817.54}_{S-PV(Div)} + \underbrace{112.31}_{P} = \underbrace{\frac{870}{1.0042^{\frac{22}{12}}}}_{PV\,(K)} + \mathbf{C}$$

and then we solve for C:

$$C = 817.54 + 112.31 - \frac{870}{1.0042^{\frac{22}{12}}} = 66.51$$

The price of a December 2017 OEX call option with a strike price of 870 is 66.51.

Exercise 20.20

In mid-February 2016, European-style options on the S&P index (OEX) expiring in December 2017 were priced as follows:

December 2017 OEX Options		
Strike Price	Call Price	Put Price
840	88.00	?
860	76.30	102.21
880	?	111.56

Given an interest rate of 0.40 % (0.004) for a December 2017 maturity (22 months in the future), use put-call parity (with dividends) to determine:

A. The price of a December 2017 OEX put option with a strike price of 840.

By the law of one price, we have that

$$S + P = PV(K) + C$$

where S is the stock price, P is the put price, K is the strike price of the option (the price you want to ensure that the stock will not drop below), and C is the call price.

In the case a stock pays dividends, however, the equation is as follows:

$$S - PV(Div) + P = PV(K) + C$$

We want to find the price of a put options (P) with a strike price of 840. By rearranging the put-call parity we get the following:

$$P = PV(K) + C - S + PV(Div)$$

With a strike price of 840 we know from the table that the price of a call option (C) is 88.00. However, in order to use the put-call parity with dividends, we also need to know the stock price minus the dividends. The first step is therefore to solve for $S - PV(Div)$.

Step 1: Solve for $S - PV(Div)$

Solving for $S - PV(Div)$ gives us:

$$S - PV(Div) + P = PV(K) + C$$

$$S - PV(Div) = PV(K) + C - P$$

Now, we can use the information from the 860 calls and puts to solve for $S - PV(Div)$.

$$S - PV(Div) = PV(K) + C - P$$

$$S - PV(Div) = \frac{860}{1.004^{\frac{22}{12}}} + 76.30 - 102.21 = 827.82$$

Step 2: Solve for the 840 put

We use the put-call parity with dividends

$$S - PV(Div) + \boldsymbol{P} = PV(K) + C$$

and then we plug in the information we have, namely

$$\underbrace{827.82}_{S-PV(Div)} + \boldsymbol{P} = \underbrace{\frac{840}{1.004^{\frac{22}{12}}}}_{PV\ (K)} + \underbrace{88.00}_{C}$$

Then we solve for P:

$$P = \frac{840}{1.004^{\frac{22}{12}}} + 88.00 - 827.82 = 94.05$$

The price of a December 2017 OEX put option with a strike price of 840 is 94.05.

B. The price of a December 2017 OEX call option with a strike price of 880.

To calculate the 880 call we again use the put-call parity with dividends:

$$S - PV(Div) + P = PV(K) + \boldsymbol{C}$$

From question A we know that $S - PV(Div)$ is equal to 827.82. With a strike price of 880 the price of a put option (P) is 111.56. Now we can plug this information in the formula

$$\underbrace{827.82}_{S-PV(Div)} + \underbrace{111.56}_{P} = \underbrace{\frac{880}{1.004^{\frac{22}{12}}}}_{PV\ (K)} + \boldsymbol{C}$$

and then we solve for C:

$$C = 827.82 + 111.56 - \frac{880}{1.004^{\frac{22}{12}}} = 65.80$$

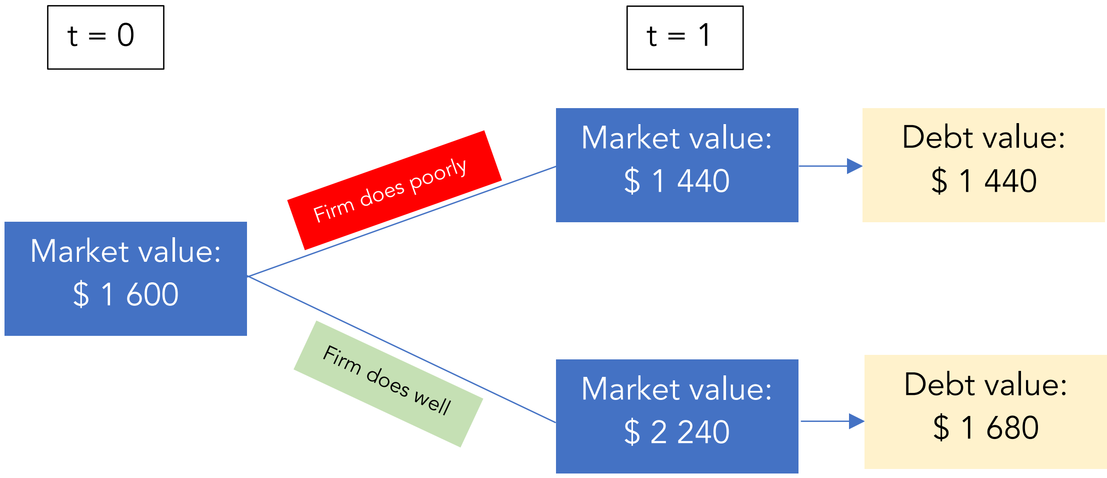
The price of a December 2017 OEX call option with a strike price of 880 is 65.80.

Chapter 21: Option valuation

Example

Hema Corp. is an all-equity firm with a current market value of $1 600 million and will be worth $1 440 million or $2 240 million in one year. The risk-free interest rate is 5 %. Suppose Hema Corp. issues zero-coupon, one-year debt with a face-value of $1 680 million and uses the proceeds to pay a special dividend to shareholders. Assuming perfect capital markets, use the binomial model to answer the following:

A. What are the payoffs of the firm's debt in one year?



The payoffs of the firm's debt in one year will be either $ 1 680 million (if the firm does well) or $1 440 million if the firm does poorly and defaults. Remember that equity holders are residual claimers, as the box below explains.

> If a company goes into liquidation, all of its assets are distributed to its creditors. Secured creditors are first in line. Next are unsecured creditors, including employees who are owed money. Stockholders are paid last.

B. What is the value of the of the debt today?

We can use the binomial model. To compute the delta (Δ), use the following formula:

$$\Delta = \frac{C_u - C_d}{S_u - S_d}$$

where Δ is the sensitivity of option price to debt value, C_u is the call option value if the firm does well, C_d is the put option value if the firm does poorly and defaults, S_u is the value of the firm if it does well, and S_d is the value of the firm if it does poorly.

Therefore,

$$\Delta = \frac{\$1\ 680\ million - \$1\ 440\ million}{\$2\ 240\ million - \ \$1\ 440\ million} = 0.3$$

The delta of the option is 0.3.

To calculate the risk-free investment in the replicating portfolio, use the following formula:

$$B = \frac{C_d\ - S_d * \Delta}{1 + r_f}$$

where B is the risk-free investment in the replicating portfolio, C_d is the put option value if the firm does poorly and defaults, S_d is the value of the firm if it does poorly, Δ is the sensitivity of option price to debt value and r_f is the risk-free rate. Therefore,

$$B = \frac{\$1\ 440\ million - \$1\ 440\ million * 0.3}{1.05} = \$960\ million$$

The risk-free investment in the replicating portfolio is $960 million.

To determine the value of the debt today, use the following formula:

$$C = \Delta * S + B$$

where C is the value of the debt (option), Δ is the sensitivity of option price to debt value, S is the firm's current market value, and B is the risk-free investment in the replicating portfolio.

Therefore,

$$C = 0.3 * \$1\,600\,million + \$960\,million = \$1\,440\,million$$

The value of the debt today is $1 440 million.

C. What is the yield on the debt?

To calculate the yield on the debt, we use the formula:

$$Yield = \frac{Debt - Value\ of\ debt}{Value\ of\ debt} = \frac{Debt}{Value\ of\ debt} - 1$$

$$Yield = \frac{\$1\,680\ - \$1\,440}{\$1\,440} = 0.1667$$

The yield of the debt is 16.67 %.

D. Using Modigliani-Miller, what is the value of Hema's equity before the dividend is paid? What is the value of equity just after the dividend is paid?

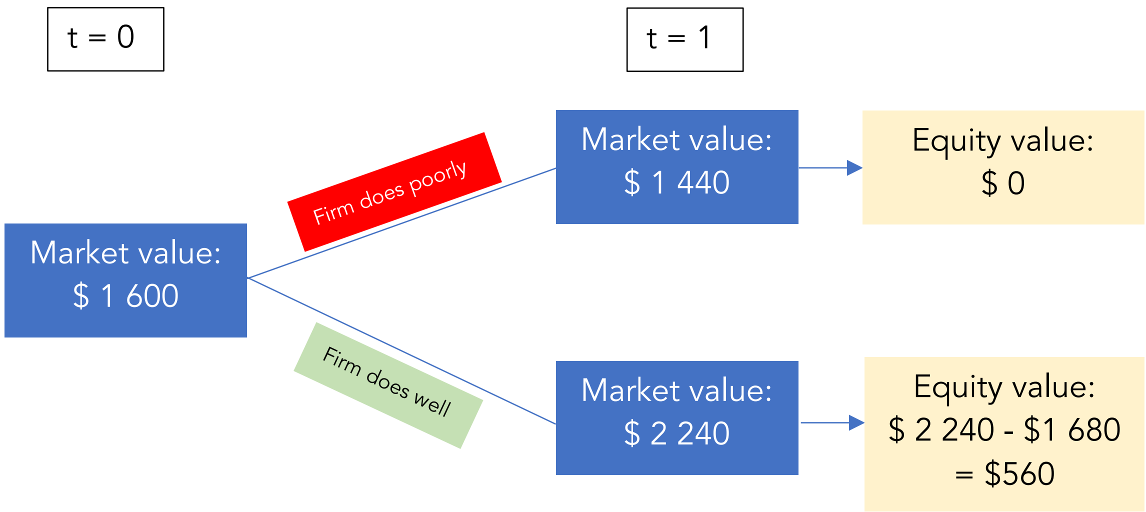
According to Modigliani-Miller, the initial value should not change ($1 600 million). To determine the ex-dividend value, subtract the dividend (debt value) from the initial value, as in the following equation:

$$Ex - dividend\ value = Initial\ market\ value - Debt\ value$$
$$Ex - dividend\ value = \$1\,600\,million - \$1\,440\,million = \$160\,million$$

So, the value of Hema's equity before the dividend is paid the initial value of $1 600 million, while the value of equity just after the dividend is paid is $160 million.

E. Show that the ex-dividend value of Hema's equity is consistent with the binomial model. What is the Δ of the equity, when viewed as a call option on the firm's assets?

First, we calculate the equity payoffs, C_u and C_d .



Therefore

$$C_u = \underbrace{\$2\ 240}_{\text{Market value in one year}} - \underbrace{\$1\ 680}_{\text{Debt value in one year}} = \$560$$

$$C_d = \underbrace{0}$$

If the firm does poorly and defaults, the market value of $1 440 million is distributed to debt holders. Therefore equity payoff is 0.

To compute the delta (Δ), use the following formula:

$$\Delta = \frac{C_u - C_d}{S_u - S_d}$$

where Δ is the sensitivity of option price to debt value, C_u is the put option value if the firm does well, C_d is the put option value if the firm does poorly and defaults, S_u is the value of the firm if it does well, and S_d is the value of the firm if it does poorly.

Therefore,

$$\Delta = \frac{\$560\ million - \$0}{\$2\ 240\ million - \ \$1\ 440\ million} = 0.7$$

To calculate the risk-free investment in the replicating portfolio, use the following formula:

$$B = \frac{C_d \ - S_d * \Delta}{1 + r_f}$$

where B is the risk-free investment in the replicating portfolio, C_d is the call option value if the firm does poorly and defaults, S_d is the value of the firm if it does poorly, Δ is the sensitivity of option price to debt value and r_f is the risk-free rate. Therefore,

$$B = \frac{\$0 - \$1\ 440\ million * 0.7}{1.05} = -\$960\ million$$

To determine the value of the equity, use the following formula:

$$C = \Delta * S + B$$

where C is the value of the equity (option), Δ is the sensitivity of option price to debt value, S is the firm's market value, and B is the risk-free investment in the replicating portfolio. Therefore,

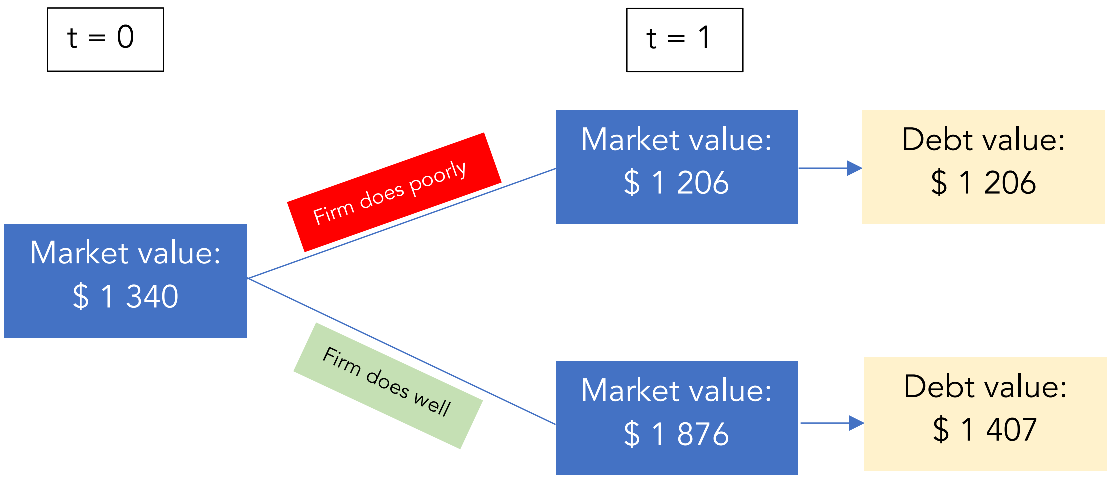
$$C = 0.7 * \$1\ 600 - \$960 = \$160$$

The delta (Δ) of the equity is 0.7, which yields a value of the equity of $160 million.

Exercise 21.9

Hema Corp. is an all-equity firm with a current market value of $1 340 million and will be worth $1 206 million or $1 876 million in one year. The risk-free interest rate is 5 %. Suppose Hema Corp. issues zero-coupon, one-year debt with a face-value of $1 407 million and uses the proceeds to pay a special dividend to shareholders. Assuming perfect capital markets, use the binomial model to answer the following:

A. What are the payoffs of the firm's debt in one year?



The payoffs of the firm's debt in one year will be either $ 1 407 million (if the firm does well) or $1 206 million if the firm does poorly and defaults. Remember that equity holders are residual claimers, as the box below explains.

> If a company goes into liquidation, all of its assets are distributed to its creditors. Secured creditors are first in line. Next are unsecured creditors, including employees who are owed money. Stockholders are paid last.

B. What is the value of the of the debt today?

We can use the binomial model. To compute the delta (Δ), use the following formula:

$$\Delta = \frac{C_u - C_d}{S_u - S_d}$$

where Δ is the sensitivity of option price to debt value, C_u is the call option value if the firm does well, C_d is the put option value if the firm does poorly and defaults, S_u is the value of the firm if it does well, and S_d is the value of the firm if it does poorly.

Therefore,

$$\Delta = \frac{\$1\,407\,million - \$1\,206\,million}{\$1\,876\,million - \$1\,206 million} = 0.3$$

The delta of the option is 0.3.

To calculate the risk-free investment in the replicating portfolio, use the following formula:

$$B = \frac{C_d - S_d * \Delta}{1 + r_f}$$

where B is the risk-free investment in the replicating portfolio, C_d is the put option value if the firm does poorly and defaults, S_d is the value of the firm if it does poorly, Δ is the sensitivity of option price to debt value and r_f is the risk-free rate. Therefore,

$$B = \frac{\$1\,206\,million - \$1\,206\,million * 0.3}{1.05} = \$804\,million$$

The risk-free investment in the replicating portfolio is $804 million.

To determine the value of the debt today, use the following formula:

$$C = \Delta * S + B$$

where C is the value of the debt (option), Δ is the sensitivity of option price to debt value, S is the firm's current market value, and B is the risk-free investment in the replicating portfolio.

Therefore,

$$C = 0.3 * \$1\,340\, million + \$804\, million = \$1\,206\, million$$

The value of the debt today is \$1 206 million.

C. What is the yield on the debt?

To calculate the yield on the debt, we use the formula:

$$Yield = \frac{Debt - Value\ of\ debt}{Value\ of\ debt} = \frac{Debt}{Value\ of\ debt} - 1$$

$$Yield = \frac{\$1\,407 - \$1\,206}{\$1\,206} = 0.1667$$

The yield of the debt is 16.67 %.

D. Using Modigliani-Miller, what is the value of Hema's equity before the dividend is paid? What is the value of equity just after the dividend is paid?

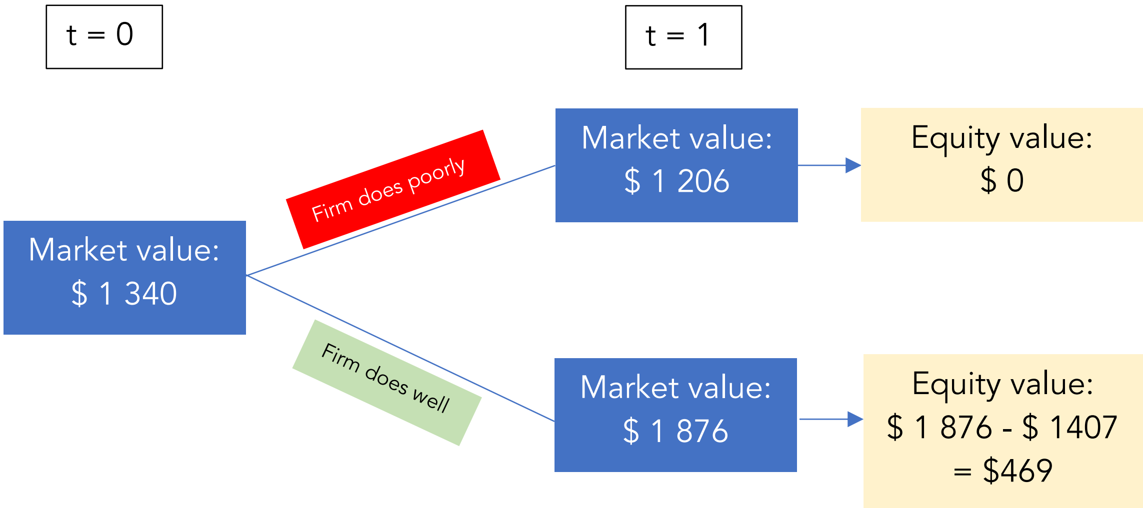
According to Modigliani-Miller, the initial value should not change (\$1 340 million). To determine the ex-dividend value, subtract the dividend (debt value) from the initial value, as in the following equation:

$$Ex - dividend\ value = Initial\ market\ value - Debt\ value$$
$$Ex - dividend\ value = \$1\,340\, million - \$1\,206\, million = \$134\, million$$

So, the value of Hema's equity before the dividend is paid the initial value of \$1 340 million, while the value of equity just after the dividend is paid is \$134 million.

E. Show that the ex-dividend value of Hema's equity is consistent with the binomial model. What is the Δ of the equity, when viewed as a call option on the firm's assets?

First, we calculate the equity payoffs, C_u and C_d .



Therefore

$$C_u = \$1\,876 - \$1\,407 = \$469$$

Market value in one year ($1 876); Debt value in one year ($1 407)

$$C_d = 0$$

If the firm does poorly and defaults, the market value of $1 876 million is distributed to debt holders. Therefore equity payoff is 0.

To compute the delta (Δ), use the following formula:

$$\Delta = \frac{C_u - C_d}{S_u - S_d}$$

where Δ is the sensitivity of option price to debt value, C_u is the put option value if the firm does well, C_d is the put option value if the firm does poorly and defaults, S_u is the value of the firm if it does well, and S_d is the value of the firm if it does poorly.

Therefore,

$$\Delta = \frac{\$469\ million - \$0}{\$1\,876\ million - \ \$1\,206\ million} = 0.7$$

To calculate the risk-free investment in the replicating portfolio, use the following formula:

$$B = \frac{C_d - S_d * \Delta}{1 + r_f}$$

where B is the risk-free investment in the replicating portfolio, C_d is the call option value if the firm does poorly and defaults, S_d is the value of the firm if it does poorly, Δ is the sensitivity of option price to debt value and r_f is the risk-free rate. Therefore,

$$B = \frac{\$0 - \$1\,206\,million * 0.7}{1.05} = -\$804\,million$$

To determine the value of the equity, use the following formula:

$$C = \Delta * S + B$$

where C is the value of the equity (option), Δ is the sensitivity of option price to debt value, S is the firm's market value, and B is the risk-free investment in the replicating portfolio. Therefore,

$$C = 0.7 * \$1\,340 - \$804 = \$134$$

The delta (Δ) of the equity is 0.7, which yields a value of the equity of $134 million.

Chapter 22: Real options

Example

Suppose the current value of a dealership is $5.12 million and the first-year free cash flow is expected to be $512 000. Assume that the beta of the dealership is 1.9 and that the appropriate cost of capital for this investment is 11.17 %. Also consider that the volatility and the risk-free rate are 45 % and 5.2 %, respectively.

A. What is the beta of a corporation whose only assets in a one-year option to open a dealership?

First, we must compute the current value of the asset **without** the dividends that will be missed:

$$S^x = S - PV(Div)$$

$$S^x = \$5.12\ million - \frac{\$0.512}{1.117} = \$4.6634\ million$$

Then we calculate the present value of the value of the dealership:

$$PV(K) = \frac{Cost}{1 + r_f}$$

Certain cost is discounted with the risk free rate

Note that we discount with the risk-free rate, because the cost of the dealership is certain.

$$PV(K) = \frac{\$5.12\ million}{1.052} = \$4.8669$$

Next, we calculate the value of the call option to open the dealership. According to Black-Scholes, the price of a call option on a non-dividend paying stock is:

$$C = S * N(d_1) - PV(K) * N(d_2)$$

where S is the current price of the stock, K is the exercise price, and $N(d)$ is the cumulative normal distribution, that is the probability that an outcome from a standard normal distribution will be below a certain value.

Note that is a stock pays a dividend, then we replace S by S^x where

S^x = S – PV(Div)

In the context of real options, the dividend represents the free cash flow lost from delay.

We have already calculated S^x (current market value of asset without free cash flow lost from delay), and PV(K) (present value of the cost of the dealership). However, we need to calculate the values of d_1 and d_2.

We calculate d_1 and d_2 as follows:

$$d_1 = \frac{ln\left[\frac{S^x}{PV(K)}\right]}{\sigma * \sqrt{T}} + \frac{\sigma * \sqrt{T}}{2}$$

$$d_2 = d_1 - \sigma * \sqrt{T}$$

where σ is the annual volatility, and T is the number of years left to expiration.

Here the values of d_1 and d_2 are:

$$d_1 = \frac{ln\left[\frac{4.6634}{4.8669}\right]}{0.45 * \sqrt{\frac{365}{365}}} + \frac{0.45 * \sqrt{\frac{365}{365}}}{2} = 0.1301$$

$$d_2 = 0.1301 - 0.45 * \sqrt{\frac{365}{365}} = -0.3199$$

Substituting into the Black-Scholes formula, we get

$$C = S^x * N(d_1) - PV(K) * N(d_2)$$

$$C = \$4.6634 * N(0.1301) - \$4.8669 * N(-0.3199)$$

$$C = \$4.6634 * 0.5518 - 4.8669 * 0.3745 = \$0.7506\, million$$

The beta is

$$\beta_C = \frac{\Delta S}{\Delta S + B} * \beta_S = \frac{S^x * N(d_1)}{C} * \beta_{dealership}$$

$$\beta = \frac{\$4.6634\, million * 0.5518}{\$0.7506\, million} * 1.9 = 6.51$$

B. What is the beta if the first year's cash flows are expected to be $704 000, so a working dealership is worth $7.04 million?

If a working dealership is worth $7.04 million, the current value of the asset without the dividends that will be missed is:

$$S^x = S - PV(Div)$$

$$S^x = \$7.04\ million - \frac{\$0.704}{1.117} = \$6.4133\ million$$

Next, we calculate the value of the call option to open the dealership using Black-Scholes:

$$C = S^x * N(d_1) - PV(K) * N(d_2)$$

We have already calculated S^x (current market value of asset without free cash flow lost from delay), and PV(K) (present value of the cost of the dealership). However, we need to calculate the values of d_1 and d_2.

$$d_1 = \frac{ln\left[\frac{S^x}{PV(K)}\right]}{\sigma * \sqrt{T}} + \frac{\sigma * \sqrt{T}}{2}$$

$$d_1 = \frac{ln\left[\frac{6.4133}{4.8669}\right]}{0.45 * \sqrt{\frac{365}{365}}} + \frac{0.45 * \sqrt{\frac{365}{365}}}{2} = 0.8331$$

$$d_2 = d_1 - \sigma * \sqrt{T}$$

$$d_2 = 0.8331 - 0.45 * \sqrt{\frac{365}{365}} = 0.3881$$

Substituting into the Black-Scholes formula, we get

$$C = S^x * N(d_1) - PV(K) * N(d_2)$$

$$C = \$6.4133 * N(0.8331) - \$4.8669 * N(0.3881)$$

$$C = \$4.6634 * 0.799 - 4.8669 * 0.651 = \$0.5412\ million$$

The beta is

$$\beta_C = \frac{\Delta S}{\Delta S + B} * \beta_S = \frac{S^x * N(d_1)}{C} * \beta_{dealership}$$

$$\beta = \frac{\$6.4133\ million * 0.799}{\$0.5412\ million} * 1.9 = 17.99$$

Exercise 22.9

Suppose the current value of a dealership is \$5.2 million and the first-year free cash flow is expected to be \$520 000. Assume that the beta of the dealership is 2 and that the appropriate cost of capital for this investment is 12 %. Also consider that the volatility and the risk-free rate are 40 % and 5 %, respectively.

A. What is the beta of a corporation whose only assets in a one-year option to open a dealership?

First, we must compute the current value of the asset **without** the dividends that will be missed:

$$S^x = S - PV(Div)$$

$$S^x = \$5.2\ million - \frac{\$0.520}{1.12} = \$4.74\ million$$

Then we calculate the present value of the value of the dealership:

$$PV(K) = \frac{Cost}{1 + r_f}$$

Certain cost is discounted with the risk free rate

Note that we discount with the risk-free rate, because the cost of the dealership is certain.

$$PV(K) = \frac{\$5.2\ million}{1.05} = \$4.9524$$

Next, we calculate the value of the call option to open the dealership. According to Black-Scholes, the price of a call option on a non-dividend paying stock is:

$$C = S * N(d_1) - PV(K) * N(d_2)$$

where S is the current price of the stock, K is the exercise price, and $N(d)$ is the cumulative normal distribution, that is the probability that an outcome from a standard normal distribution will be below a certain value.

Note that is a stock pays a dividend, then we replace S by S^x where

S^x = S – PV(Div)

In the context of real options, the dividend represents the free cash flow lost from delay.

We have already calculated S^x (current market value of asset without free cash flow lost from delay), and PV(K) (present value of the cost of the dealership). However, we need to calculate the values of d_1 and d_2.

We calculate d_1 and d_2 as follows:

$$d_1 = \frac{ln\left[\frac{S^x}{PV(K)}\right]}{\sigma * \sqrt{T}} + \frac{\sigma * \sqrt{T}}{2}$$

$$d_2 = d_1 - \sigma * \sqrt{T}$$

where σ is the annual volatility, and T is the number of years left to expiration.

Here the values of d_1 and d_2 are:

$$d_1 = \frac{ln\left[\frac{4.74}{4.9524}\right]}{0.40 * \sqrt{\frac{365}{365}}} + \frac{0.40 * \sqrt{\frac{365}{365}}}{2} = 0.19$$

$$d_2 = 0.19 - 0.40 * \sqrt{\frac{365}{365}} = -0.21$$

Substituting into the Black-Scholes formula, we get

$$C = S^x * N(d_1) - PV(K) * N(d_2)$$

$$C = \$4.47 * N(0.19) - \$4.9524 * N(-0.21) = \$0.74\, million$$

The beta is

$$\beta_C = \frac{\Delta S}{\Delta S + B} * \beta_S = \frac{S^x * N(d_1)}{C} * \beta_{dealership}$$

$$\beta = \frac{\$4.74\, million * 0.57}{\$0.74\, million} * 2 = 7.35$$

B. What is the beta if the first year's cash flows are expected to be \$710 000, so a working dealership is worth \$7.1 million?

If a working dealership is worth \$7.1 million, the current value of the asset without the dividends that will be missed is:

$$S^x = S - PV(Div)$$

$$S^x = \$7.1\, million - \frac{\$0.710}{1.12} = \$6.47\, million$$

Next, we calculate the value of the call option to open the dealership using Black-Scholes:

$$C = S^x * N(d_1) - PV(K) * N(d_2)$$

We have already calculated S^x (current market value of asset without free cash flow lost from delay), and PV(K) (present value of the cost of the dealership). However, we need to calculate the values of d_1 and d_2.

$$d_1 = \frac{ln\left[\frac{S^x}{PV(K)}\right]}{\sigma * \sqrt{T}} + \frac{\sigma * \sqrt{T}}{2}$$

$$d_1 = \frac{ln\left[\frac{6.47}{4.9524}\right]}{0.40 * \sqrt{\frac{365}{365}}} + \frac{0.40 * \sqrt{\frac{365}{365}}}{2} = 0.8683$$

$$d_2 = d_1 - \sigma * \sqrt{T}$$

$$d_2 = 0.8683 - 0.40 * \sqrt{\frac{365}{365}} = 0.4683$$

Substituting into the Black-Scholes formula, we get

$$C = S^x * N(d_1) - PV(K) * N(d_2)$$

$$C = \$6.47 * N(0.8683) - \$4.9524 * N(0.4683) = \$1.855\ million$$

The beta is

$$\beta_C = \frac{\Delta S}{\Delta S + B} * \beta_S = \frac{S^x * N(d_1)}{C} * \beta_{dealership}$$

$$\beta_C = \frac{\$6.47\ million * 0.8074}{\$1.855\ million} * 2 = 5.63$$

Chapter 28: Mergers and acquisitions

Example

Your company has earnings per share of $5. It has 1 million shares outstanding, each of which has a price of $45. You are thinking of buying TargetCo, which has earnings per share of $3. TargetCo has 1 million shares outstanding, and a price per share of $30. You will pay for TargetCo by issuing new shares. There are no expected synergies from this transaction. Suppose you offer an exchange ratio such that, at current pre-announcement share prices for both firms, the offer represents at 15 % premium to buy TargetCo. Assume that on the announcement the target price will go up and your price will go down to reflect the fact that you are willing to pay a premium for TargetCo. Assume that the takeover will occur with certainty and all market participants know this on the announcement of the takeover.

A. What is the price per share of the combined corporation **immediately** after the merger is completed?

A 15 % premium means that I will have to pay 15 % more per share to buy TargetCo, which initially had a price per share of $30. Therefore, the price per TargetCo share after the merger is:

$$\$30 * 1.15 = \$34.5$$

TargetCo has 1 million shares outstanding, so the total price of TargetCo is:

$$\$34.5 * 1\ million = \$34.5\ million$$

Each share in my company has a price of $45. Thus, I will have to issue the following number of shares in order to buy TargetCo:

$$Numbers\ of\ shares\ issued = \frac{Total\ amount\ I\ have\ to\ finance}{\underbrace{Price\ per\ share}}$$

What each share is worth

$$= \frac{\$34.5\ million}{\$45} = 0.767\ million$$

Since 0.767 million new shares will be issued, the share price will be:

$$Share\ price = \frac{Enterprise\ value\ (combined\ corporation)}{Number\ of\ shares}$$

$$Share\ price = \frac{\overbrace{(\$45 * 1\ million\ shares)}^{\text{My company}} + \overbrace{(\$30 * 1\ million\ shares)}^{\text{TargetCo}}}{\underbrace{1\ million + 0.767\ million}_{\text{The new number of shares in my company}}} = \$42.44$$

The price per share of the combined corporation immediately after the merger is completed is $42.44.

B. What is the price of your company **immediately** after the announcement?

Same as the price after the merger: $42.44 per share

C. What is the price of TargetCo **immediately** after the announcement?

When my company buys TargetCo, TargetCo receives 0.767 million shares (numbers of shares in order to buy TargetCo) with a price per share of $42.44. That means that TargetCo shareholders receive:

$$0.767\ million\ shares * \$42.44\ per\ share = \$32.55\ million$$

There are 1 million TargetCo shares outstanding, so the share price of TargetCo will be:

$$Share\ price = \frac{Enterprise\ value}{Number\ of\ shares\ oustanding} = \frac{\$32.55\ million}{1\ million} = \$32.55$$

D. What is the actual premium your company will pay?

The premium will be:

$$Premium = \frac{Price\ per\ share\ paid}{Current\ price\ per\ share}$$

$$Premium = \frac{\$32.55\ per\ share}{\$30\ per\ share} = 0.085 = 8.5\ \%$$

Exercise 28.14

Your company has earnings per share of \$4. It has 1.6 million shares outstanding, each of which has a price of \$56. You are thinking of buying TargetCo, which has earnings per share of \$1. TargetCo has 1.3 million shares outstanding, and a price per share of \$24. You will pay for TargetCo by issuing new shares. There are no expected synergies from this transaction. Suppose you offer an exchange ratio such that, at current pre-announcement share prices for both firms, the offer represents at 20 % premium to buy TargetCo. Assume that on the announcement the target price will go up and your price will go down to reflect the fact that you are willing to pay a premium for TargetCo. Assume that the takeover will occur with certainty and all market participants know this on the announcement of the takeover.

A. What is the price per share of the combined corporation **immediately** after the merger is completed?

A 20 % premium means that I will have to pay 20 % more per share to buy TargetCo, which initially had a price per share of \$24. Therefore, the price per TargetCo share after the merger is:

$$\$24 * 1.20 = \$28.8$$

TargetCo has 1.3 million shares outstanding, so the total price of TargetCo is:

$$\$28.8 * 1.3\ million = \$37.44\ million$$

Each share in my company has a price of \$56. Thus, I will have to issue the following number of shares in order to buy TargetCo:

$$Numbers\ of\ shares\ issued = \frac{Total\ amount\ I\ have\ to\ finance}{\underbrace{Price\ per\ share}_{\text{What each share is worth}}}$$

$$= \frac{\$37.44\ million}{\$56} = 0.669\ million$$

Since 0.669 million new shares will be issued, the share price will be:

$$Share\ price = \frac{Enterprise\ value\ (combined\ corporation)}{Number\ of\ shares}$$

$$Share\ price = \frac{\overbrace{(\$56 * 1.6\ million\ shares)}^{\text{My company}} + \overbrace{(\$24 * 1.3\ million\ shares)}^{\text{TargetCo}}}{\underbrace{1.6\ million + 0.669\ million}_{\text{The new number of shares in my company}}} = \$53.24$$

The price per share of the combined corporation immediately after the merger is completed is $53.24.

B. What is the price of your company **immediately** after the announcement?

Same as the price after the merger: $53.24 per share

C. What is the price of TargetCo **immediately** after the announcement?

When my company buys TargetCo, TargetCo receives 0.669 million shares (numbers of shares in order to buy TargetCo) with a price per share of $53.24. That means that TargetCo shareholders receive:

$$0.669\ million\ shares * \$53.24\ per\ share = \$35.62\ million$$

There are 1.3 million TargetCo shares outstanding, so the share price of TargetCo will be:

$$Share\ price = \frac{Enterprise\ value}{Number\ of\ shares\ oustanding} = \frac{\$35.62\ million}{1.3\ million} = \$27.40$$

D. What is the actual premium your company will pay?

The premium will be:

$$Premium = \frac{Price\ per\ share\ paid - Current\ price\ per\ share}{Current\ price\ per\ share}$$

$$Premium = \frac{\$27.4\ per\ share - \$24\ per\ share}{\$24\ per\ share} = 0.1417 = 14.17\ \%$$

Chapter 30: Risk management

Example

You have been hired as a risk manager for Acorn Savings and Loan. Currently, Acorn's balance sheet is as follows (in millions of dollars):

Assets		Liabilities	
Cash reserves	51.2	Checking and savings	78.7
Auto loans	100.7	Certificates of deposit	98.1
Mortgages	152.4	Long-term financing	106.7
		Total liabilities	283.5
		Owner's equity	20.8
Total assets	304.3	Total liabilities and equity	304.3

When you analyze the duration of loans, you find that the duration of the auto loans is 2.2 years, while the mortgages have a duration of 6.9 years. Both the cash reserves and the checking and savings accounts have a zero duration. The CDs have a duration of 2.4 years, and the long-term financing has a 10.4-year duration.

A. What is the duration of Acorn's equity?

Duration is a measure of the sensitivity of the price of a bond or other debt instrument to a change in interest rates.

Certain factors can affect a bond's duration, including:

- **Time to maturity.** The longer the maturity, the higher the duration, and the greater the interest rate risk. Consider two bonds that each yield 5% and cost $1,000 but have different maturities. A bond that matures faster – say, in one year – would repay its true cost faster than a bond that matures in 10 years. Consequently, the shorter-maturity bond would have a lower duration and less risk.

- **Coupon rate.** A bond's coupon rate is a key factor in calculation duration. If we have two bonds that are identical with the exception on their coupon rates, the bond with the higher coupon rate will pay back its original costs faster than the bond with a lower yield. The higher the coupon rate, the lower the duration, and the lower the interest rate risk.

The duration of a portfolio of assets A and B is given by:

$$D_{A+B} = \frac{A}{A+B} * D_A + \frac{B}{A+B} * D_B$$

The duration of assets is:

$$D_{Assets} = \underbrace{\frac{51.2}{304.3} * 0\ years}_{\text{Cash reserves}} + \underbrace{\frac{100.7}{304.3} * 2.2\ years}_{\text{Auto loans}} + \underbrace{\frac{152.4}{304.3} * 6.9\ years}_{\text{Mortgages}} = 4.18\ years$$

The duration of the liabilities is:

$$D_{Liabilities} = \underbrace{\frac{78.7}{283.5} * 0\ years}_{\text{Checking and savings}} + \underbrace{\frac{98.1}{283.5} * 2.4\ years}_{\text{CDs}} + \underbrace{\frac{106.7}{283.5} * 10.4\ years}_{\text{Long term financing}} = 4.74\ years$$

The duration of the equity is:

$$D_{Equity} = \underbrace{\frac{304.3\ million}{20.8\ million} * 4.18\ years}_{\text{Assets}} - \underbrace{\frac{283.5\ million}{20.8\ million} * 4.74\ years}_{\text{Liabilities}} = -3.45\ years$$

B. Suppose Acorn experiences a rash of mortgage prepayments, reducing the size of the mortgage portfolio from \$152.4 million to \$101.6 million, and increasing cash reserves to \$102 million. What is the duration of Acorn's equity now? If interest rates are currently 4 % and were to fall to 3 %, estimate the approximate change in the value of Acorn's equity. Assume interest rate are APRs based on monthly compounding.

Acorn's balance is now:

Assets		Liabilities	
Cash reserves	102	Checking and savings	78.7
Auto loans	100.7	Certificates of deposit	98.1
Mortgages	101.6	Long-term financing	106.7
		Total liabilities	283.5
		Owner's equity	20.8
Total assets	304.3	Total liabilities and equity	304.3

The duration of assets is:

$$D_{Assets} = \underbrace{\frac{102}{304.3} * 0\ years}_{\text{Cash reserves}} + \underbrace{\frac{100.7}{304.3} * 2.2\ years}_{\text{Auto loans}} + \underbrace{\frac{101.6}{304.3} * 6.9\ years}_{\text{Mortgages}} = 3.03\ years$$

The duration of the equity is:

$$D_{Equity} = \underbrace{\frac{304.3\ million}{20.8\ million} * 3.03\ years}_{\text{Assets}} - \underbrace{\frac{283.5\ million}{20.8\ million} * 4.74\ years}_{\text{Liabilities}} = -20.28\ years$$

If interest rates drop by 1 %, we would expect the value of Acorn's equity to drop by about:

$$Percent\ change\ in\ value \approx -Duration * \frac{\varepsilon}{1 + \frac{r}{k}}$$

where ε is the change in interest rates, r is the interest rate before the change, and k is the number of periods in a year. Then we get the following:

$$Percent\ change\ in\ value \approx\ -(-20.28) * \frac{-1}{1+\frac{0.04}{12}} =\ -20.21\ \%$$

C. Suppose that after the prepayments in part B. but before a change in interest rates, Acorn considers managing its risk by selling mortgages and/or buying 10-year Treasury STRIPS (zero coupon bonds). How many should the firm buy or sell to eliminate its current interest rate risk?

Acorn would like to increase the duration of its assets, so it should use cash to buy long-term bonds. Because 10-year STRIPS (zero coupon bonds) have a 10-year duration, we can use:

$$Amount\ to\ exchange = \frac{Change\ in\ portfolio\ duration * \overbrace{Portfolio\ value}^{\text{Equity}}}{Change\ in\ asset\ duration}$$

$$Amount\ to\ exchange\ = \frac{20.28\ years * \$20.8}{10\ years} = \$42.18\ million$$

That is, we should buy $42.18 million worth of 10-year STRIPS.

Exercise 30.12

You have been hired as a risk manager for Acorn Savings and Loan. Currently, Acorn's balance sheet is as follows (in millions of dollars):

Assets		Liabilities	
Cash reserves	51.3	Checking and savings	79.8
Auto loans	98.1	Certificates of deposit	100.2
Mortgages	151.3	Long-term financing	99.4
		Total liabilities	279.4
		Owner's equity	21.3
Total assets	300.7	Total liabilities and equity	300.7

When you analyze the duration of loans, you find that the duration of the auto loans is 2.1 years, while the mortgages have a duration of 7.2 years. Both the cash reserves and the checking and savings accounts have a zero duration. The CDs have a duration of 1.9 years, and the long-term financing has a 9.2-year duration.

A. What is the duration of Acorn's equity?

The duration of a portfolio of assets A and B is given by:

$$D_{A+B} = \frac{A}{A+B} * D_A + \frac{B}{A+B} * D_B$$

The duration of assets is:

$$D_{Assets} = \underbrace{\frac{51.3}{300.7} * 0\ years}_{\text{Cash reserves}} + \underbrace{\frac{98.1}{300.7} * 2.1\ years}_{\text{Auto loans}} + \underbrace{\frac{151.3}{300.7} * 7.2\ years}_{\text{Mortgages}} = 4.308\ years$$

The duration of the liabilities is:

$$D_{Liabilities} = \underbrace{\frac{79.8}{279.4} * 0\ years}_{\text{Checking and savings}} + \underbrace{\frac{100.2}{279.4} * 1.9\ years}_{\text{CDs}} + \underbrace{\frac{99.4}{279.4} * 9.2\ years}_{\text{Long term financing}} = 3.954 years$$

The duration of the equity is:

$$D_{Equity} = \underbrace{\frac{300.7\ million}{21.3\ million} * 4.308\ years}_{\text{Assets}} - \underbrace{\frac{279.4\ million}{21.3\ million} * 3.954\ years}_{\text{Liabilities}} = 8.95\ years$$

B. Suppose Acorn experiences a rash of mortgage prepayments, reducing the size of the mortgage portfolio from $151.3 million to $100.9 million, and increasing cash reserves to $101.7 million. What is the duration of Acorn's equity now? If interest rates are currently 4 % and were to fall to 3 %, estimate the approximate change in the value of Acorn's equity. Assume interest rate are APRs based on monthly compounding.

Acorn's balance is now:

Assets		Liabilities	
Cash reserves	101.7	Checking and savings	79.8
Auto loans	98.1	Certificates of deposit	100.2
Mortgages	100.9	Long-term financing	99.4
		Total liabilities	279.4
		Owner's equity	21.3
Total assets	300.7	Total liabilities and equity	300.7

The duration of assets is:

$$D_{Assets} = \underbrace{\frac{101.7}{300.7} * 0\ years}_{\text{Cash reserves}} + \underbrace{\frac{98.1}{300.7} * 2.1\ years}_{\text{Auto loans}} + \underbrace{\frac{100.9}{300.7} * 7.2\ years}_{\text{Mortgages}} = 3.101\ years$$

The duration of the equity is:

$$D_{Equity} = \underbrace{\frac{300.7\ million}{21.3\ million} * 3.101\ years}_{\text{Assets}} - \underbrace{\frac{279.4\ million}{21.3\ million} * 3.954\ years}_{\text{Liabilities}} = -8.092\ years$$

If interest rates drop by 1 %, we would expect the value of Acorn's equity to drop by about:

$$Percent\ change\ in\ value \approx -Duration * \frac{\varepsilon}{1 + \frac{r}{k}}$$

where ε is the change in interest rates, r is the interest rate before the change, and k is the number of periods in a year. Then we get the following:

$$Percent\ change\ in\ value \approx -(-8.092) * \frac{-1}{1 + \frac{0.04}{12}} = -8.07\ \%$$

C. Suppose that after the prepayments in part B. but before a change in interest rates, Acorn considers managing its risk by selling mortgages and/or buying 10-year Treasury STRIPS (zero coupon bonds). How many should the firm buy or sell to eliminate its current interest rate risk?

Acorn would like to increase the duration of its assets, so it should use cash to buy long-term bonds. Because 10-year STRIPS (zero coupon bonds) have a 10-year duration, we can use:

$$Amount\ to\ exchange = \frac{Change\ in\ portfolio\ duration * \overbrace{Portfolio\ value}^{\text{Equity}}}{Change\ in\ asset\ duration}$$

$$Amount\ to\ exchange = \frac{8.092\ years * \$21.3}{10\ years} = \$17.236\ million$$

That is, we should buy $17.236 million worth of 10-year STRIPS.