FIE402: Workshop 2

Chapter 17: Payout policy

Example

Raviv Industries has $225 million in cash that it can use for a share repurchase. Suppose instead Raviv invests the fund in an account paying 14 % interest for one year.

A. It the corporate tax rate is 46 %, how much additional cash will Raviv have at the end of the year net of corporate taxes?

To calculate the additional cash, use the following formula:

$$Additional\ cash = Cash * Interest\ rate * (1 - \tau_c)$$

Therefore,

$$Additional\ cash = \$225\ million * 0.14 * (1 - 0.46) = \$17.010\ million$$

If the corporate tax is 46 %, the additional cash will be $17.010 million.

B. If investors pay a 30 % tax rate on capital gains, by how much will the value of their shares have increased, net of capital gains taxes?

To compute the increased value of the shares, use the following formula:

$$Increased\ value = Additional\ cash * (1 - \tau_g)$$

Therefore,

$$Increased\ value = \$17.010\ million * (1 - 0.30) = \$11.907\ million$$

If investors pay a 30 % tax rate on capital gains, the value of their shares will increase by $11.907 million.

C. If investors pay a 38 % tax rate on interest income, how much would they have had if they invested the $225 million on their own?

To calculate the amount investors would have if they had invested the $225 million on their own, use the following formula:

$$Amount = Cash * Interest\ rate * (1 - \tau_i)$$

Therefore,

$$Amount = \$225\ million * 0.14 * (1 - 0.38) = \$19.530\ million$$

Investors will have $19.530 million.

D. Suppose Raviv retained the cash so that it would not need to raise new funds from outside investors for an expansion it has planned for next year. If it did raise new funds, it would have to pay issuance fees. How much does Raviv need to save in issuance fees to make retaining the cash beneficial for its investors? Assume fees can be expensed for corporate tax purposes.

Here we have to different options:

Option 1: Pay out the cash to investors. They invest the cash on their own, but they pay tax on interest income.

Option 2: Retain the cash, meaning that the cash stays in the firm. Then we don’t need to pay issuance fees.

If we choose Option 1, we must pay issuance fees for raising new funds to finance the expansion next year. Remember that issuance fees are tax deductible.

To calculate the amount that Raviv needs to save in issuance fees, we first calculate the after tax-cost of **retaining** the cash:

$$After - tax\ cost\ of\ retaining\ cash = \underbrace{\$19.530\ million}_{\text{How much investors get when they invest cash on their own, after interest income tax.}} - \underbrace{\$11.907\ million}_{\text{How much investors get when Raviv invests cash in an account, after corporate tax and capital gains tax.}} = \$7.623\ million$$

The after-tax cost of retaining the cash is $7.623 million.

Next, to compute what a $1 spent on fees is equivalent to for the investors after corporate and capital gain tax, using the following formula:

$$Investors\ Portion\ of\ Fees = \$1 * (1 - \tau_c) * (1 - \tau_g)$$

Therefore,

$$Investors\ Portion\ of\ Fees = \$1 * (1 - 0.46) * (1 - 0.30) = \$0.378$$

For the investors, $1 spent on fees is equivalent to $0.378.

To calculate the amount that must be saved in issuance fees to make retaining the cash beneficial for its investors, use the following formula:

$$Fees = \frac{After - tax\ Cost\ of\ Retaining\ Cash}{Investor\ Portion\ of\ Fees}$$

Therefore,

$$Fees = \frac{(\$\ 19.530\ million - \$11.907\ million)}{\$0.378} = \$20.167\ million$$

The amount that Raviv needs to save in issuance fees is $20.167 million.

Exercise 17.26

Raviv Industries has $100 million in cash that it can use for a share repurchase. Suppose instead Raviv invests the fund in an account paying 10 % interest for one year.

A. It the corporate tax rate is 40 %, how much additional cash will Raviv have at the end of the year net of corporate taxes?

To calculate the additional cash, use the following formula:

$$Additional\ cash = Cash * Interest\ rate * (1\ - \tau_c)$$

Therefore,

$$Additional\ cash = \$100\ million * 0.10 * (1\ - 0.40) = \$6\ million$$

If the corporate tax is 40 %, the additional cash will be $6 million.

B. If investors pay a 20 % tax rate on capital gains, by how much will the value of their shares have increased, net of capital gains taxes?

To compute the increased value of the shares, use the following formula:

$$Increased\ value = Additional\ cash * (1 - \tau_g)$$

Therefore,

$$Increased\ value = \$6\ million * (1 - \ 0.20) = \$4.8\ million$$

If investors pay a 20 % tax rate on capital gains, the value of their shares will increase by $4.8 million.

C. If investors pay a 30 % tax rate on interest income, how much would they have had if they invested the $100 million on their own?

To calculate the amount investors would have if they had invested the $100 million on their own, use the following formula:

$$Amount = Cash * Interest\ rate * (1 - \tau_i)$$

Therefore,

$$Amount = \$100 million * 0.10 * (1 - \ 0.30) = \$7\ million$$

Investors will have $7 million.

D. Suppose Raviv retained the cash so that it would not need to raise new funds from outside investors for an expansion it has planned for next year. If it did raise new funds, it would have to pay issuance fees. How much does Raviv need to save in issuance fees to make retaining the cash beneficial for its investors? Assume fees can be expensed for corporate tax purposes.

Here we have to different options:

> Option 1: Pay out the cash to investors. They invest the cash on their own, but they pay tax on interest income.
>
> Option 2: Retain the cash, meaning that the cash stays in the firm. Then we don't need to pay issuance fees.

If we choose Option 1, we must pay issuance fees for raising new funds to finance the expansion next year. Remember that issuance fees are tax deductible.

To calculate the amount that Raviv needs to save in issuance fees, we first calculate the after tax-cost of **retaining** the cash:

$$After - tax\ cost\ of\ retaining\ cash = \underbrace{\$7\ million}_{\substack{\text{How much investors get} \\ \text{when they invest cash} \\ \text{on their own, after} \\ \text{interest income tax.}}} - \underbrace{\$4.8 million}_{\substack{\text{How much investors get} \\ \text{when Raviv invests cash in an} \\ \text{account, after corporate tax} \\ \text{and capital gains tax.}}} = \$2.2\ million$$

The after-tax cost of retaining the cash is \$2.2 million.

Next, to compute what a \$1 spent on fees is equivalent to for the investors after corporate and capital gain tax, using the following formula:

$$Investors\ Portion\ of\ Fees = \$1 * (1 - \tau_c) * (1 - \tau_g)$$

Therefore,

$$Investors\ Portion\ of\ Fees = \$1 * (1 - 0.40) * (1 - 0.20) = \$0.48$$

For the investors, \$1 spent on fees is equivalent to \$0.48.

To calculate the amount that must be saved in issuance fees to make retaining the cash beneficial for its investors, use the following formula:

$$Fees = \frac{After - tax\ Cost\ of\ Retaining\ Cash}{Investor\ Portion\ of\ Fees}$$

Therefore,

$$Fees = \frac{(\$\,7\,million - \$4.8\,million)}{\$0.48} = \$4.583\,million$$

The amount that Raviv needs to save in issuance fees is $4.583 million.

Chapter 18: Capital budgeting and valuation with leverage

Example

You are evaluating a project that requires and investment of $112 today and guarantees a single cash flash of $135 one year from now. You decide to use 100 % debt financing, that is, you will borrow $112. The risk-free rate is 3 % and the tax rate is 42 %. Assume that the investment is fully depreciated at the end of the year, so without leverage you would owe taxes on the difference between the project cash flow and the investment, that is, $135 - $112 = $23.

A. Calculate the NPV of this investment opportunity using the APV method.

The APV (Adjusted Present Value) method consists of two steps:

Step 1: Find unlevered cash flows and then the unlevered present value of the project

Step 2: Find present value of the interest tax shield

The sum of the two equals the adjusted present value, which is the levered value of the project:

APV = PV(FCF levered) + PV(Interest tax shield)

What does the "levered value" mean? This simply means that we take into account how debt financing affects the present value of the project. In contrast, the unlevered value assumes that the project is 100 % equity financed. Therefore, the unlevered value does not include the benefits we get from paying interest on debt. Interest payments reduce earnings before tax, and therefore also reduce taxes. This reduction in taxes equals the interest tax shield. Remember that any cost savings count as a payment!

In order to find the **net present value** of the project, we must subtract the investment value.

Step 1: Find unlevered cash flow and the unlevered value of the project

In Step 1 we don't take into account the interest tax shield, only the unlevered cash flows. We pretend the project is 100 % equity financed.

What are the cash flows in this project?

Depreciation is not a cash flow but affects the taxable amount. Since taxes are a cash flow, depreciation has an indirect effect on cash flows.

	Year 0	Year 1
Investment	$-\$112$	
Cash flow		$\$135$
Depreciation		$-\$112$
Earnings before tax		$\$23$
Tax, 42 %		$(\$135 - \$112) * 0.42 = \$9.66$
Cash flow after tax, FCF	$-\$112$	$\$125.34$

To calculate the unlevered free cash flow (FCF) at the end of the year after taxes, we use the following formula:

$$FCF_{AT} = FCF - (FCF - Investment) * Tax\ rate$$

Therefore,

$$FCF_{AT} = \$135 - (\$135 - \$112) * 0.42 = \$125.34$$

The after-tax free cash flow at the end of the year is $125.34.

To compute the unlevered value of the project, use the following formula:

$$V^U = \frac{FCF}{(1 + r_U)}$$

where

- V^U is the unlevered value of the project
- FCF is the free cash flow
- r_U is the unlevered cost of capital, here equal to the risk-free interest rate

Therefore,

$$V^U = \frac{\$125.34}{1.03} = \$121.69$$

The unlevered value of the project is $121.69.

Step 2: Find the present value of the interest tax shield

To calculate the present value of the interest tax shield, use the following formula:

$$PV(Interest\ tax\ shield) = \frac{\tau_c * r_D * D}{(1 + r_D)}$$

where

- $PV(Interest\ tax\ shield)$ is the present value of the interest tax shield
- τ_c is the marginal corporate tax
- r_D is the cost of debt, here equal to the risk-free interest rate
- D is the value of the debt

Therefore,

$$PV(Interest\ tax\ shield) = \frac{0.42 * 0.03 * \$112}{1.03} = \$1.37$$

The present value of the tax shield is $1.37.

Now that we have completed the two steps, we can find the adjusted present value:

$$APV = V^L = V^U + PV(Interest\ tax\ shield)$$

Therefore,

$$V^L = \$121.69 + \$1.37 = \$123.06$$

The last step is to calculate the NPV, by subtracting the investment value:

$$NPV = V^L - Investment$$

Then we get:

$$NPV = \$123.06 - \$112 = \$11.06$$

The NPV of this investment opportunity is $11.06.

B. Using your answer to part A, calculate the WACC of the project.

To calculate the WACC of the project, use the following formula:

$$r_{WACC} = r_U - \left[\frac{D}{V^L} * \tau_c * \ r_D * \frac{1 + r_U}{1 + \ r_D}\right]$$

Note that here we have that $r_U = r_f$ and $r_D = \ r_f$.

Therefore,

$$r_{WACC} = \ 0.03 - \frac{\$112}{\$123.06} * \ 0.03 * \ 0.42 = 0.0185 = 1.85\ \%$$

The WACC of the project is 1.85 %.

C. Verify that you get the same answer using the WACC method to calculate NPV.

The WACC method consists of two steps:

Step 1: Find the unlevered cash flows

Step 2: Find the present value of the levered project by discounting unlevered cash flows with the WACC

When using the WACC method, we calculate the levered value of the project directly by discounting the free cash flows with the WACC, as shown below:

$$V^L = \frac{FCF}{(1 + r_{WACC})}$$

Therefore,

$$V^L = \frac{\$125.34}{1.0185} = \$123.06$$

Then we find the net present value by subtracting the investment:

$$NPV = V^L - Investment$$

$$NPV = \$123.06 - \$112 = \$11.06$$

The NPV based on the levered value of the project is \$11.06, exactly the same as we calculated in question A using the APV method. The two methods should always give the same answer!

D. Finally, show that flow-to-equity method also correctly gives the NPV of the investment opportunity.

The flow-to-equity method consists of two steps:

Step 1: Find the project's free cash flow to equity (FCFE)

Step 2: Find the present value of the project by discounting the free cash flow to equity with the cost of equity

Step 1: Find the project's free cash flow to equity (FCFE)

The first step in the FTE method is to determine the project's free cash flow to equity (FCFE).

The FCFE is the free cash flow that remains after adjusting for interest payments, debt issuance and debt repayment. To calculate the FCFE, use the following formula:

$$FCFE = FCF \underbrace{- (1 - \tau_c) * Interest\ payments}_{} + Debt\ issuance - Debt\ repayment$$

Because of taxes we only pay $(1 - \tau_c)$ in interest payment.

What are the cash flows?

	Year 0	Year 1
Investment	$-\$112$	
Cash flow		$\$135$
Depreciation		$-\$112$
Earnings before tax		$\$23$
Tax, 42 %		$(\$135 - \$112) * 0.42 = \$9.66$
Cash flow after tax, FCF	$-\$112$	$\$125.34$
$-(1-\tau_c) * Interest\ payments$		$-(1-0.42) * 0.03 * \$112 = -1.95$
+ Borrowings	$+\$112$	$-\$112$
FCFE, Free cash flow to equity	$\$0$	$\$11.39$

The free cash flow-to-equity for year 0 is:

$$FCFE_0 = -\$112 + \$112 = \$0$$

The free cash flow-to-equity for year 1 is:

$$FCFE_1 = (\$125.34) - [(1-0.42) * 0.03 * \$112] - (\$112) = \$11.39$$

Step 2: Find the present value of the project by discounting FCFE

To calculate the value to equity, use the following formula:

$$Value\ to\ equity = FCFE_0 + \frac{FCFE_1}{(1 + r_E)}$$

Note that here we have that $r_E = r_D$ because the cash flow in year 1 is certain.

Therefore,

$$Value\ to\ equity = \$0 + \frac{\$11.39}{1.03} = \$11.06$$

Exercise 18.16

You are evaluating a project that requires and investment of $108 today and guarantees a single cash flash of $118 one year from now. You decide to use 100 % debt financing, that is, you will borrow $108. The risk-free rate is 4 % and the tax rate is 33 %. Assume that the investment is fully depreciated at the end of the year, so without leverage you would owe taxes on the difference between the project cash flow and the investment, that is, $118 - $108 = $10.

A. Calculate the NPV of this investment opportunity using the APV method.

The APV (Adjusted Present Value) method consists of two steps:

Step 1: Find unlevered cash flows and then the unlevered present value of the project

Step 2: Find present value of the interest tax shield

The sum of the two equals the adjusted present value, which is the levered value of the project:

APV = PV(FCF levered) + PV(Interest tax shield)

What does the "levered value" mean? This simply means that we take into account how debt financing affects the present value of the project. In contrast, the unlevered value assumes that the project is 100 % equity financed. Therefore, the unlevered value does not include the benefits we get from paying interest on debt. Interest payments reduce earnings before tax, and therefore also reduce taxes. This reduction in taxes equals the interest tax shield. Remember that any cost savings count as a payment!

In order to find the **net present value** of the project, we must subtract the investment value.

Step 1: Find unlevered cash flow and the unlevered value of the project

Under Step 1 we don't take into account the interest tax shield, only the unlevered cash flows. We pretend the project is 100 % equity financed.

What are the cash flows in this project?

Depreciation is not a cash flow but affects the taxable amount. Since taxes are a cash flow, depreciation has an indirect effect on cash flows.

	Year 0	Year 1
Investment	$-$ \$108	
Cash flow		\$118
Depreciation		$-$ \$108
Earnings before tax		\$10
Tax, 33 %		$(\$118 - \$108) * 0.33 = \$3.3$
Cash flow after tax, FCF	$-$ \$108	\$114.7

To calculate the unlevered free cash flow (FCF) at the end of the year after taxes, we use the following formula:

$$FCF_{AT} = FCF - (FCF - Investment) * Tax\ rate$$

Therefore,

$$FCF_{AT} = \$118 - (\$118 - \$108) * 0.33 = \$114.7$$

The after-tax free cash flow at the end of the year is \$114.7.

To compute the unlevered value of the project, use the following formula:

$$V^U = \frac{FCF}{(1 + r_U)}$$

Note that here we have that $r_U = r_f$.

where

- V^U is the unlevered value of the project

- FCF is the free cash flow
- r_U is the unlevered cost of capital, here equal to the risk free rate.

Therefore,

$$V^U = \frac{\$114.7}{1.04} = \$110.29$$

The unlevered value of the project is $110.29.

Step 2: Find the present value of the interest tax shield

To calculate the present value of the interest tax shield, use the following formula:

$$PV(Interest\ tax\ shield) = \frac{\tau_c * r_D * D}{(1 + r_D)}$$

where

- $PV(Interest\ tax\ shield)$ is the present value of the interest tax shield
- τ_c is the marginal corporate tax
- r_D is the cost of debt, here equal to the risk-free interest rate
- D is the value of the debt

Therefore,

$$PV(Interest\ tax\ shield) = \frac{0.33 * 0.04 * \$108}{1.03} = \$1.37$$

The present value of the tax shield is $1.37.

Now that we have completed the two steps, we can find the adjusted present value:

$$APV = V^L = V^U + PV(Interest\ tax\ shield)$$

Therefore,

$$V^L = \$110.29 + \$1.37 = \$111.66$$

The last step is to calculate the NPV, by subtracting the investment value:

$$NPV = V^L - Investment$$

Then we get:

$$NPV = \$111.66 - \$108 = \$3.66$$

The NPV of this investment opportunity is $3.66.

B. Using your answer to part A, calculate the WACC of the project.

To calculate the WACC of the project, use the following formula:

$$r_{WACC} = r_U - \left[\frac{D}{V^L} * \tau_c * \ r_D * \frac{1 + r_U}{1 + \ r_D}\right]$$

Note that here we have that $r_U = r_f$ and $r_D = \ r_f$.

Therefore,

$$r_{WACC} = \ 0.04 - \frac{\$108}{\$110.29} * \ 0.33 * \ 0.04 * \frac{1.04}{1.04} \ = 0.0271 = 2.71\ \%$$

The WACC of the project is 2.71 %.

C. Verify that you get the same answer using the WACC method to calculate NPV.

The WACC method consists of two steps:

Step 1: Find the unlevered cash flows

Step 2: Find the present value of the levered project by discounting unlevered cash flows with the WACC

When using the WACC method, we calculate the levered value of the project directly by discounting the free cash flows with the WACC, as shown below:

$$V^L = \frac{FCF}{(1 + r_{WACC})}$$

Therefore,

$$V^L = \frac{\$114.7}{1.0271} = \$111.66$$

Then we find the net present value by subtracting the investment:

$$NPV = V^L - Investment$$

$$NPV = \$111.66 - \$108 = \$3.66$$

The NPV based on the levered value of the project is $11.06, exactly the same as we calculated in question A using the APV method. The two methods should always give the same answer!

D. Finally, show that flow-to-equity method also correctly gives the NPV of the investment opportunity.

The flow-to-equity method consists of two steps:

Step 1: Find the project's free cash flow to equity (FCFE)

Step 2: Find the present value of the project by discounting the free cash flow to equity with the cost of equity

Step 1: Find the project's free cash flow to equity (FCFE)

The first step in the FTE method is to determine the project's free cash flow to equity (FCFE).

The FCFE is the free cash flow that remains after adjusting for interest payments, debt issuance and debt repayment. To calculate the FCFE, use the following formula:

$$FCFE = FCF - \underbrace{(1 - \tau_c) * Interest\ payments}_{\text{Because of taxes we only pay } (1-\tau_c) \text{ in interest payment.}} + Debt\ issuance - Debt\ repayment$$

What are the cash flows?

	Year 0	Year 1
Investment	$-\$108$	
Cash flow		$\$118$
Depreciation		$-\$108$
Earnings before tax		$\$10$
Tax, 33 %		$(\$118 - \$108) * 0.33 = \$3.3$
Cash flow after tax, FCF	$-\$108$	$\$114.7$
$-(1-\tau_c) * Interest\ payments$		$-(1-0.33) * 0.04 * \$108 = -2.89$
+ Borrowings	$+\$108$	$-\$108$
FCFE, Free cash flow to equity	$\$0$	$\$3.81$

The free cash flow-to-equity for year 0 is:

$$FCFE_0 = -\$108 + \$108 = \$0$$

The free cash flow-to-equity for year 1 is:

$$FCFE_1 = (\$114.7) - [(1-0.33) * 0.04 * \$108] - (\$108) = \$3.81$$

Step 2: Find the present value of the project by discounting FCFE

To calculate the value to equity, use the following formula:

$$Value\ to\ equity = FCFE_0 + \frac{FCFE_1}{(1 + r_E)}$$

Note that here we have that $r_E = r_D$ because the cash flow in year 1 is certain.

Therefore,

$$Value\ to\ equity = \$0 + \frac{\$3.81}{1.04} = \$3.66$$

Summary

	Method 1: APV	Method 2: WACC	Method 3: FCFE
Cash flow	PV(FCF unlevered) + PV(Interest tax shield)	Unlevered FCF	Levered FCF → Free cash flow to equity
Discount rate	Unlevered (pretax cost of capital) → Pretax WACC	After tax WACC	Cost of equity

Chapter 25: Leasing

Example

Consider a 7-year lease for a \$350 000 bottling machine, with a residual market value of \$122 500 at the end 7 years. If the risk-free interest rate is 6.2 % APR (annual percentage rate) with monthly compounding, compute the **monthly** lease payment in a perfect market for the following leases:

A. A fair market value lease

First, we need to calculate the monthly interest rate since we have monthly compounding.

What is the monthly interest rate?

$$Monthly\ interest\ rate = \frac{Annual\ interest\ rate}{12\ months} = \frac{0.062}{12} = 0.00517 = 0.517\ \%$$

Then we need to find the number of months included in the lease contract.

How many months are there in 7 years?

$$Number\ of\ months = 7\ years * 12\ months = 84\ months$$

Next, we need to know this relationship:

$$PV(Lease\ payments) = Purchase\ price - PV(Residual\ value)$$

Therefore,

$$PV(Lease\ payments) = \$350\ 000 - \frac{\$122\ 500}{(1 + 0.00517)^{84}} = \$270\ 562$$

So, know we know the present value of all the lease payments. However, we are interested in the monthly lease payment. It is crucial to know that the first lease payment is paid up front (in period 0), and the remaining 83 monthly payments are paid as an annuity. Therefore, we can use the following annuity formula, where L is the monthly lease payment:

$$\$270\ 564 = L * \left[1 + \frac{1}{0.00517} * \left(1 - \frac{1}{1.00517^{83}}\right)\right]$$

Solving for L we get that

$$L = \$3\ 959$$

(n-1) because first payment is up front and therefore is not discounted!

B. A \$1.00 out lease

In this case the lessor will only receive \$1.00 at the conclusion of the lease. Therefore, the present value of the lease payments should be \$350 000, where L is the monthly lease payment:

$$\$350\ 000 = L * \left[1 + \frac{1}{0.00517} * \left(1 - \frac{1}{1.00517^{83}}\right)\right]$$

Solving for L we get that

$$L = \$5\ 121$$

C. A fixed price lease with an \$72 000 final price.

In this case, the lessor will receive \$72 000 at the conclusion of the lease. Thus,

$$PV(Lease\ payments) = Purchase\ price - PV(Residual\ value)$$

$$PV(Lease\ payments) = \$350\ 000 - \frac{\$72\ 000}{(1 + 0.00517)^{84}} = \$303\ 311$$

Again, because the first lease payment is paid up front, and the remaining 83 payments are paid as an annuity we can use the annuity formula, where L is the monthly lease payment:

$$\$303\ 311 = L * \left[1 + \frac{1}{0.00517} * \left(1 - \frac{1}{1.00517^{83}}\right)\right]$$

Solving for L we get that

$$L = \$4\ 438$$

Exercise 25.3

Consider a 6-year lease for a \$350 000 bottling machine, with a residual market value of \$122 500 at the end 7 years. If the risk-free interest rate is 5.9 % APR (annual percentage rate) with monthly compounding, compute the **monthly** lease payment in a perfect market for the following leases:

A. A fair market lease

First, we need to calculate the monthly interest rate since we have monthly compounding.

What is the monthly interest rate?

$$Monthly\ interest\ rate = \frac{Annual\ interest\ rate}{12\ months} = \frac{0.059}{12} = 0.00492 = 0.492\ \%$$

Then we need to find the number of months included in the lease contract.

How many months are there in 6 years?

$$Number\ of\ months = 6\ years * 12\ months = 72\ months$$

Next, we need to know this relationship:

$$PV(Lease\ payments) = Purchase\ price - PV(Residual\ value)$$

Therefore,

$$PV(Lease\ payments) = \$350\ 000 - \frac{\$122\ 500}{(1+0.00492)^{72}} = \$263\ 945.7$$

So, know we know the present value of all the lease payments. However, we are interested in the monthly lease payment. It is crucial to know that the first lease payment is paid up front (in period 0), and the remaining 83 monthly payments are paid as an annuity. Therefore, we can use the following annuity formula, where L is the monthly lease payment:

$$\$263\ 945.7 = L * \left[1 + \frac{1}{0.00492} * \left(1 - \frac{1}{1.00492^{71}}\right)\right]$$

Solving for L we get that

$$L = \$4\ 340.55$$

(n-1) because first payment is up front and therefore is not discounted!

B. A \$1.00 out lease

In this case the lessor will only receive \$1.00 at the conclusion of the lease. Therefore, the present value of the lease payments should be \$350 000, where L is the monthly lease payment:

$$\$350\ 000 = L * \left[1 + \frac{1}{0.00492} * \left(1 - \frac{1}{1.00492^{71}}\right)\right]$$

Solving for L we get that

$$L = \$5\ 755.7$$

C. A fixed price lease with an $50 000 final price.

In this case, the lessor will receive $50 000 at the conclusion of the lease. Thus,

$$PV(Lease\ payments) = Purchase\ price - PV(Residual\ value)$$

$$PV(Lease\ payments) = \$350\ 000 - \frac{\$50\ 000}{(1+0.00492)^{84}} = \$314\ 875.8$$

Again, because the first lease payment is paid up front, and the remaining 71 payments are paid as an annuity we can use the annuity formula, where L is the monthly lease payment:

$$\$314\ 875.8 = L * \left[1 + \frac{1}{0.00492} * \left(1 - \frac{1}{1.00492^{71}}\right)\right]$$

Solving for L we get that

$$L = \$5\ 178.09$$

Example

Suppose Amazon is considering the purchase of computer servers and network infrastructure to expand its very successful business offering cloud-based computing. In total, it will purchase $47.3 million in new equipment. This equipment will qualify for accelerated depreciation: 20 % can be expensed immediately, followed by 32 %, 19.2 %, 11.52 %, 11.52 % and 5.57 % over the next five years.

However, because of the firm's substantial loss carryforwards and other credits, Amazon estimates its marginal tax rate will be 10 % over the next five years, so it will get very little tax benefit from the depreciation expenses. Thus, Amazon considers leasing the equipment instead. Suppose Amazon and the lessor face the same 7.8 % borrowing rate, but the lessor has a 35 % tax rate. For the purpose of this question, assume the equipment is worthless after five years, the lease term is five years, and the lease qualifies as a true tax lease.

A. What is the lease rate for which the lessor will break even?

That is, the lessor buys the equipment and then lends the equipment out to Amazon. We need to find the lease rate that makes the lessor break even.

First, we compute the FCF from buying the equipment. The FCF are shown below:

Buy	0	1	2	3	4	5
Capital expenditure	$-\$47.3\ m$					
Depreciation	$0.2 * \$47.3\ m = \$9.46\ m$	$0.32 * \$47.3\ m = \$15.136\ m$	$0.192 * \$47.3\ m = \$9.08\ m$	$0.1152 * \$47.3\ m = \$5.45\ m$	$0.1152 * \$47.3\ m = \$5.45\ m$	$0.0576 * \$47.3\ m = \$2.72\ m$
Depreciation tax shield Tax rate = 35 %	$0.35 * \$9.46\ m = \$3.311\ m$	$0.35 * \$15.136\ m = \$5.23\ m$	$0.35 * \$9.08\ m = \$3.72\ m$	$0.35 * \$5.45\ m = \$1.907\ m$	$0.35 * \$5.45\ m = \$1.907\ m$	$0.35 * \$2.72\ m = \$0.954\ m$
Free cash flow (buy)	$-\$43.989\ m$	$\$5.298\ m$	$\$3.179\ m$	$\$1.907\ m$	$\$1.907\ m$	$\$0.954\ m$

The discount rate we must use is the borrowing rate after tax:

$$Borrowing\ rate\ after\ tax = 0.078 * (1 - 0.35) = 0.0507 = 5.07\ \%$$

The NPV of the FCF from buying the machine is ($ million):

$$NPV = -\$43.989 + \frac{\$5.298}{1.057} + \frac{\$3.179}{1.057^2} + \frac{\$1.907}{1.057^3} + \frac{\$1.907}{1.057^4} + \frac{\$0.954}{1.057^5} = -\$32.113$$

Therefore, to break even, the present value of the after-tax lease payments must equal $32.113 million. We solve for the lease payment L:

$$\$32.113\ million = L * (1 - 0.35) * \left[1 + \frac{1}{0.0507} * \left(1 - \frac{1}{1.0507^4}\right)\right]$$

$$L = \$10.882\ million$$

B. What is the gain to Amazon with this lease rate?

To compute this amount, we first compute the FCF from buying and then from leasing the machine from Amazon's perspective. Remember that Amazon's tax rate is 10 %.

Step 1: FCF from buying

Buy	0	1	2	3	4	5
Capital expenditure	$-\$47.3\ m$					
Depreciation	$0.2 * \$47.3\ m = \$9.46\ m$	$0.32 * \$47.3\ m = \$15.136\ m$	$0.192 * \$47.3\ m = \$9.08\ m$	$0.1152 * \$47.3\ m = \$5.45\ m$	$0.1152 * \$47.3\ m = \$5.45\ m$	$0.0576 * \$47.3\ m = \$2.72\ m$
Depreciation tax shield Tax rate =10 %	$0.10 * \$9.46\ m = \$0.946\ m$	$0.10 * \$15.136\ m = \$1.514\ m$	$0.10 * \$9.08\ m = \$0.908\ m$	$0.10 * \$5.45\ m = \$0.545\ m$	$0.10 * \$5.45\ m = \$0.545\ m$	$0.1 * \$2.72\ m = \$0.272\ m$
Free cash flow (buy)	$-\$46.354\ m$	$\$1.514\ m$	$\$0.908\ m$	$\$0.545\ m$	$\$0.545\ m$	$\$0.272\ m$

Step 2: FCF from leasing

Lease	0	1	2	3	4	5
Lease payments	$-\$10.882\ m$	$-\$10.882\ m$	$-\$10.882\ m$	$-\$10.882\ m$	$-\$10.882\ m$	
Income tax savings Tax rate = 10 %	$0.1 * \$10.882\ m = \$1.088\ m$	$0.1 * \$10.882\ m = \$1.088\ m$	$0.1 * \$10.882\ m = \$1.088\ m$	$0.1 * \$10.882\ m = \$1.088\ m$	$0.1 * \$10.882\ m = \$1.088\ m$	
Free cash flow (lease)	$-\$9.794\ m$	$-\$9.794\ m$	$-\$9.794\ m$	$-\$9.794\ m$	$-\$9.794\ m$	0

Step 3: Lease vs. buy

Lease vs. buy	0	1	2	3	4	5
Lease - Buy	$\$36.56\ m$	$-\$11.308\ m$	$-\$10.702\ m$	$-\$10.339\ m$	$-\$10.339\ m$	$-\$0.272\ m$

Step 4: Find the NPV of (Lease – buy)

The discount rate we must use is the borrowing rate after tax:

$$Borrowing\ rate\ after\ tax = 0.078 * (1 - 0.10) = 0.0702 = 7.02\ \%$$

The NPV of (Lease – Buy) is:

$$NPV = \$36.56\,m + \frac{-\$11.308\,m}{1.0702^1} + \frac{-\$10.702\,m}{1.0702^2} + \frac{-\$10.339\,m}{1.0702^3} + \frac{-\$10.339\,m}{1.0702^4} + \frac{-\$0.272\,m}{1.0702^5} = \$0.139\,m$$

At a lease rate of \$10.882 and a tax rate of 10 %, Amazon has a gain of \$0.139 million.

Note that if the present value of (Lease – Buy) is positive, it means that the discounted cash flows from leasing are higher than the discounted cash flows from buying. In this case, we prefer leasing.

Cash flows from buying:

Negative cash flow: Capital expenditure

Positive cash flow: Depreciation tax shield

Cash flows from leasing:

Negative cash flow: Lease payments

Positive cash flow: Income tax savings

C. What is the source of the gain in this transaction?

The source of the gain is the difference in tax rates between the two parties. Because the depreciation tax shield is more accelerated than the lease payments, there is a gain from shifting the depreciation tax shields to the party with the higher tax rate, which in this case is the lessor.

Exercise 25.10

Suppose Amazon is considering the purchase of computer servers and network infrastructure to expand its very successful business offering cloud-based computing. In total, it will purchase \$48 million in new equipment. This equipment will qualify for accelerated depreciation: 20 % can be expensed immediately, followed by 32 %, 19.2 %, 11.52 %, 11.52 % and 5.76 % over the next five years.

However, because of the firm's substantial loss carryforwards and other credits, Amazon estimates its marginal tax rate will be 10 % over the next five years, so it will get very little tax benefit from the depreciation expenses. Thus, Amazon considers leasing the equipment instead. Suppose Amazon and the lessor face the same 8 % borrowing rate, but the lessor has a 35 % tax rate. For the purpose of this question, assume the equipment is worthless after five years, the lease term is five years, and the lease qualifies as a true tax lease.

A. What is the lease rate for which the lessor will break even?

That is, the lessor buys the equipment and then lends the equipment out to Amazon. We need to find the lease rate that makes the lessor break even.

First, we compute the FCF from buying the equipment. The FCF are shown below:

Buy	0	1	2	3	4	5
Capital expenditure	$-\$48\ m$					
Depreciation	$0.2 * \$48\ m = \$9.6\ m$	$0.32 * \$48\ m = \$15.36\ m$	$0.192 * \$48\ m = \$9.216\ m$	$0.1152 * \$48\ m = \$5.53\ m$	$0.1152 * \$48\ m = \$5.53\ m$	$0.0576 * \$48\ m = \$2.765\ m$
Depreciation tax shield Tax rate = 35 %	$0.35 * \$9.6m = \$3.36\ m$	$0.35 * \$15.36m = \$5.376\ m$	$0.35 * \$9.216\ m = \$3.226\ m$	$0.35 * \$5.5296\ m = \$1.935\ m$	$0.35 * \$5.53\ m = \$1.935\ m$	$0.35 * \$2.765\ m = \$0.968\ m$
Free cash flow (buy)	$-\$44.64\ m$	$\$5.376\ m$	$\$3.226\ m$	$\$1.935\ m$	$\$1.935\ m$	$\$0.968\ m$

The discount rate we must use is the borrowing rate after tax:

$$Borrowing\ rate\ after\ tax = 0.08 * (1 - 0.35) = 0.052 = 5.2\ \%$$

The NPV of the FCF from buying the machine is ($ million):

$$NPV = -\$44.64 + \frac{\$5.376}{1.052} + \frac{\$3.226}{1.052^2} + \frac{\$1.935}{1.052^3} + \frac{\$1.935}{1.052^4} + \frac{\$0.968}{1.052^5} = -\$32.622$$

Therefore, to break even, the present value of the after-tax lease payments must equal $32.662 million. We solve for the lease payment L:

$$\$32.662\ million = L * (1 - 0.35) * \left[1 + \frac{1}{0.052} * \left(1 - \frac{1}{1.052^4}\right)\right]$$

$$L = \$11.080\ million$$

B. What is the gain to Amazon with this lease rate?

To compute this amount, we first compute the FCF from buying and then from leasing the machine from Amazon's perspective. Remember that Amazon's tax rate is 10 %.

Step 1: FCF from buying

Buy	0	1	2	3	4	5
Capital expenditure	$-\$48\ m$					
Depreciation	$0.2 * \$48\ m = \$9.6\ m$	$0.32 * \$48\ m = \$15.36\ m$	$0.192 * \$48\ m = \$9.216\ m$	$0.1152 * \$48\ m = \$5.53\ m$	$0.1152 * \$48\ m = \$5.53\ m$	$0.0576 * \$48\ m = \$2.76\ m$
Depreciation tax shield Tax rate =10 %	$0.10 * \$9.6\ m = \$0.96\ m$	$0.10 * \$15.36\ m = \$1.536\ m$	$0.10 * \$9.216\ m = \$0.922\ m$	$0.10 * \$5.53\ m = \$0.553\ m$	$0.10 * \$5.53\ m = \$0.553\ m$	$0.1 * \$2.765\ m = \$0.276\ m$
Free cash flow (buy)	$-\$47.040\ m$	$\$1.536\ m$	$\$0.922\ m$	$\$0.553\ m$	$\$0.553\ m$	$\$0.276\ m$

Step 2: FCF from leasing

Lease	0	1	2	3	4	5
Lease payments	$-\$11.080\ m$	$-\$11.080\ m$	$-\$11.080\ m$	$-\$11.080\ m$	$-\$11.080\ m$	
Income tax savings Tax rate = 10 %	$0.1 * \$11.080\ m = \$1.108\ m$	$0.1 * \$11.080\ m = \$1.108\ m$	$0.1 * \$11.080\ m = \$1.108\ m$	$0.1 * \$11.080\ m = \$1.108\ m$	$0.1 * \$11.080\ m = \$1.108\ m$	
Free cash flow (lease)	$-\$9.972\ m$	$-\$9.972\ m$	$-\$9.972\ m$	$-\$9.972\ m$	$-\$9.972\ m$	0

Step 3: Lease vs. buy

Lease vs. buy	0	1	2	3	4	5
Lease - Buy	$\$37.068\ m$	$-\$11.508\ m$	$-\$10.894\ m$	$-\$10.525\ m$	$-\$10.525\ m$	$-\$0.276\ m$

Step 4: Find the NPV of (Lease – buy)

The discount rate we must use is the borrowing rate after tax:

$$Borrowing\ rate\ after\ tax = 0.08 * (1 - 0.10) = 0.072 = 7.2\ \%$$

The NPV of (Lease – Buy) is:

$$NPV = \$37.068\,m + \frac{-\$11.508\,m}{1.072^1} + \frac{-\$10.894\,m}{1.072^2} + \frac{-\$10.525\,m}{1.072^3} + \frac{-\$10.525\,m}{1.072^4} + \frac{-\$0.276\,m}{1.072^5} = \$0.145\,m$$

At a lease rate of $11.080 and a tax rate of 10 %, Amazon has a gain of $0.145 million.

C. What is the source of the gain in this transaction?

The source of the gain is the difference in tax rates between the lessee and the lessor. Because the depreciation tax shield is more accelerated than the lease payments, there is a gain from shifting the depreciation tax shields to the party with the higher tax rate, which in this case is the lessor.

Chapter 26: Short-term financing

Example

Your firm purchases goods from its suppliers on terms of 2.0/10, net 30.

A. What is the effective annual cost to your firm if it chooses not to take the discount and makes its payment on day 30?

The effective annual cost is as follows:

$$EAR = \left[1 + \left(\frac{Discount\ rate}{1 - Discount\ rate}\right)\right]^{\frac{365\ days}{Discount\ period}} - 1$$

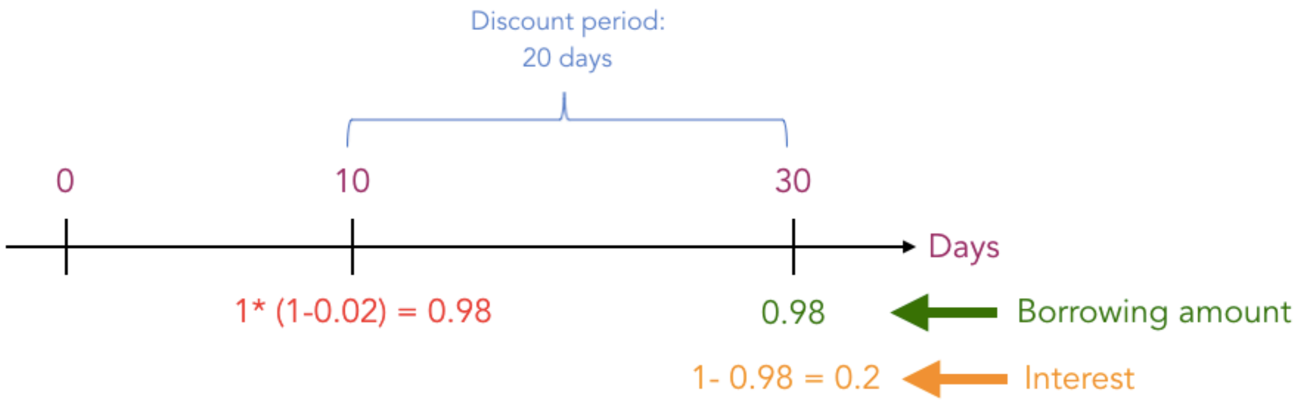
Note that "2.0/10, net 30" means that if my firm pays for goods within 10 days, we get a 2 % discount. If not, we must pay the whole amount within 30 days. The discount rate is therefore 2 %.

The discount period is the difference between the period we must pay within when we don't accept the discount (30 days) and the period we must pay within when we, in fact, accept the discount (10 days). So, the discount period is 20 days.

Then we get the following:

$$EAR = \left[1 + \left(\frac{0.02}{1 - 0.02}\right)\right]^{\frac{365\ days}{20\ days}} - 1 = 44.59\ \%$$

Alternatively, we can answer the question by drawing a time line, as shown below.



My firm is paying $2 to borrow $98 for 20 days.

The interest of 20 days is:

$$r_{20} = \frac{0.02}{0.98} = 0.02041 = 2.041\ \%$$

We need to compound this rate to an annual rate:

$$EAR = (1 + r_{20})^{\frac{365}{20}} - 1 = (1 + 0.02041)^{\frac{365}{20}} - 1 = 44.59\ \%$$

B. What is the effective annual cost to your firm if it chooses not to take the discount and makes its payment on day 40?

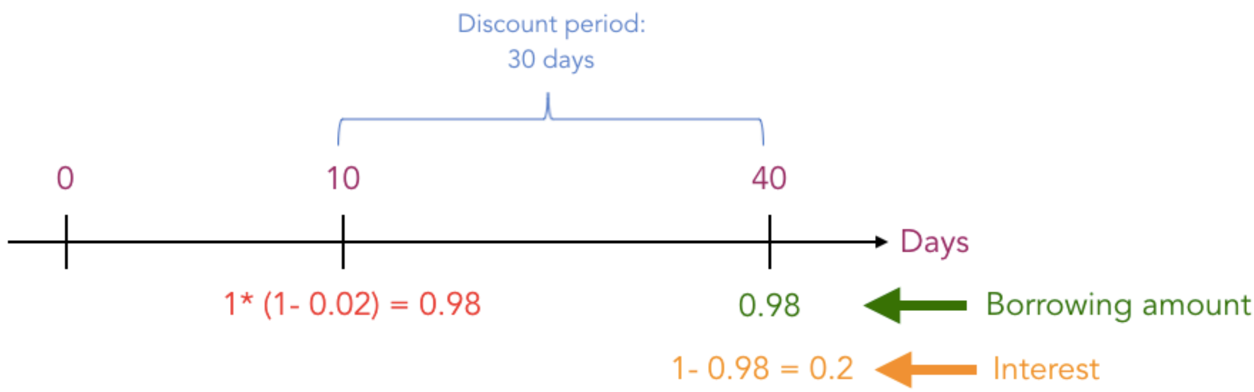
This will change from 10 days to:

$$40\ days - 10\ days = 30\ days$$

The effective annual cost decreases to the following:

$$EAR = \left[1 + \left(\frac{0.02}{1 - 0.02}\right)\right]^{\frac{365\ days}{30\ days}} - 1 = 27.86\ \%$$

Alternatively, by drawing a time line we now get:



The interest of 30 days is:

$$r_{30} = \frac{0.02}{0.98} = 0.02041 = 2.041\ \%$$

We need to compound this rate to an annual rate:

$$EAR = (1 + r_{30})^{\frac{365}{30}} - 1 = (1 + 0.02041)^{\frac{365}{30}} - 1 = 27.86\ \%$$

Note that when we compound less frequently, the effective annual interest rate decreases.

Exercise 26.10

Your firm purchases goods from its suppliers on terms of 1.7/15, net 35.

A. What is the effective annual cost to your firm if it chooses not to take the discount and makes its payment on day 35?

The effective annual cost is as follows:

$$EAR = \left[1 + \left(\frac{Discount\ rate}{1 - Discount\ rate}\right)\right]^{\frac{365\ days}{Discount\ period}} - 1$$

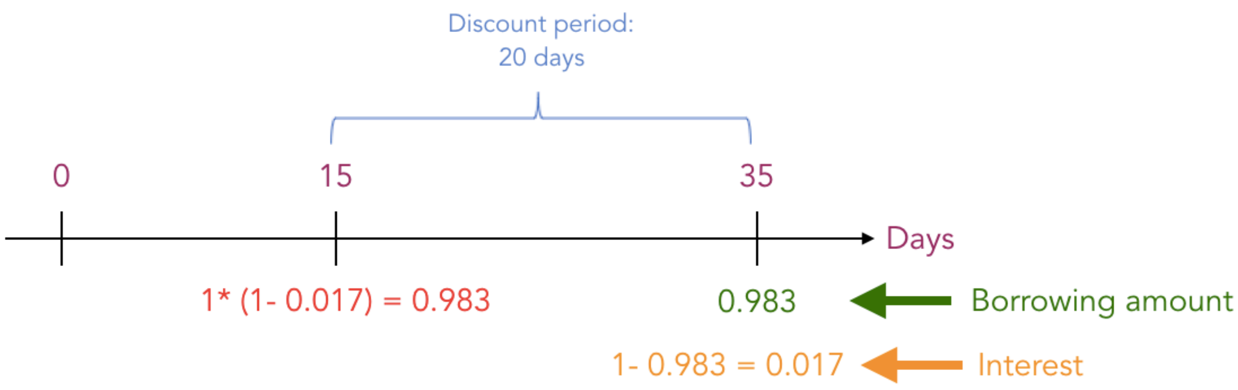
Note that "1.7/15, net 35" means that if my firm pays for goods within 15 days, we get a 1.7 % discount. If not, we must pay the whole amount within 35 days. The discount rate is therefore 1.7 %.

The discount period is the difference between the period we must pay within when we don't accept the discount (35 days) and the period we must pay within when we, in fact, accept the discount (15 days). So, the discount period is 20 days.

Then we get the following:

$$EAR = \left[1 + \left(\frac{0.017}{1 - 0.017}\right)\right]^{\frac{365\ days}{20\ days}} - 1 = 36.74\ \%$$

Alternatively, we can answer the question by drawing a time line, as shown below.



My firm is paying $1.7 to borrow $98.3 for 20 days.

The interest of 20 days is:

$$r_{20} = \frac{0.017}{0.983} = 0.0173 = 1.73\ \%$$

We need to compound this rate to an annual rate:

$$EAR = (1 + r_{20})^{\frac{365}{20}} - 1 = (1 + 0.0173)^{\frac{365}{20}} - 1 = 36.74\ \%$$

B. What is the effective annual cost to your firm if it chooses not to take the discount and makes its payment on day 45?

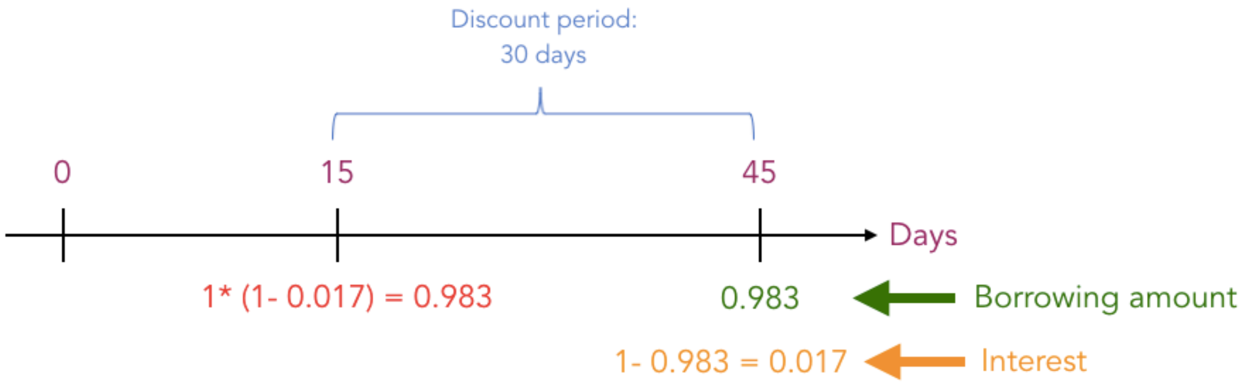
This will change from 10 days to:

$$45\ days - 15\ days = 30\ days$$

The effective annual cost decreases to the following:

$$EAR = \left[1 + \left(\frac{0.017}{1 - 0.017}\right)\right]^{\frac{365\ days}{30\ days}} - 1 = 23.2\ \%$$

Alternatively, by drawing a time line we now get:



The interest of 30 days is:

$$r_{30} = \frac{0.017}{0.983} = 0.0173 = 1.73\ \%$$

We need to compound this rate to an annual rate:

$$EAR = (1 + r_{30})^{\frac{365}{30}} - 1 = (1 + 0.0173)^{\frac{365}{30}} - 1 = 23.2\,\%$$

The effective annual rate is reduced to 23.2 % because my firm has use of money for a longer period of time.

Example

Hand-to-Mouth (H2M) is currently cash-constrained and must make a decision about whether to delay paying one of its suppliers or take out a loan. They owe the supplier \$10 500 with terms of 2.2/10 Net 40, so the suppliers will give them a 2.2 % discount if they pay by today (when the discounts period expires after 10 days). Alternatively, they can pay the full \$10 500 in one month when the invoice is due. H2M is considering three options:

Alternative A: Forgo the discount on its trade credit agreement, wait and pay the full \$10 500 in one month.

Alterative B: Borrow the money needed to pay its supplier today from Bank A, which has offered a one-month loan at an APR of 11.7 %. The bank will require a (no-interest) compensating balance of 4.8 % of the face value of the loan and will charge a \$96 loan origination fee. Because H2M has no cash, it will need to borrow the funds to cover these additional amounts as well.

Alternative C: Borrow the money needed to pay its suppliers today from Bank B, which has offered a one-month loan at an APR of 15.3 %. The loan has an 1.2 % origination fee, which again H2M need to borrow to cover.

Which alternative is the cheapest source of financing for Hand-to-Mouth?

Alternative A

The interest rate per period is calculated as follows:

$$Interest\ rate\ per\ period = \frac{Discount}{Discounted\ payment}$$

$$Interest\ rate\ per\ period = \frac{0.022 * \$10\,500}{\$10\,500 - (0.022 * \$10\,500)}$$
$$= \frac{\$231}{\$10\,500 - \$231} = 2.249\ \%$$

The loan period is one month, so the effective annual cost is:

$$EAR = (1 + Interest\ rate\ per\ period)^{number\ of\ periods\ in\ one\ year} - 1$$

$$EAR = (1 + 0.02249)^{12} - 1 = 30.59\ \%$$

Note that if we choose alternative A, we pay \$10 500 in one month, rather than paying today

$$\$\ 10\,500 * (1 - 0.022) = \$10\,269$$

Alternative B

Hand-to-Mouth will need to borrow \$10 269 to pay its supplier today, as well as the \$96 loan origination fee. The total borrowing amount is therefore:

$$\$10\,269 + \$96 = \$10\,365$$

In addition, H2M will need to meet its compensating balance requirement, which is 4.8 % of the borrowing amount. So, in order to have \$10 365 available to spend (the \$10 269 it needs to pay supplier plus the \$96 origination fee), it will have to borrow an amount that, after subtracting the compensating balance requirements, is large enough to pay the discounted payment and the origination fee:

$$Amount\ to\ borrow\ - Compensating\ balance\ requirements$$
$$= Discounted\ payment + Origination\ fee$$

$$Amount\ to\ borrow\ - \overbrace{(r * Amount\ to\ borrow)}^{\text{Compensating balance requirements}}$$
$$= Discounted\ payment + Origination\ fee$$

$$Amount\ to\ borrow\ (1 - r) = Discounted\ payment + Origination\ fee$$

$$Amount\ to\ borrow = \frac{Discounted\ payment + Origination\ fee}{(1 - r)}$$

$$Amount\ to\ borrow = \frac{\$10\ 269 + \$96}{(1 - 0.048)} = \$10\ 887.61$$

This will allow H2M to set aside 4.8 % of the loan (\$10 887.61 * 0.048 = \$522.61) and still have \$10 887.61 - \$522.61 = \$10.365 to pay the origination fee and have a net loan of \$10 269.

The bank is charging 11.7 % APR, compounded monthly, which translates into a monthly rate of:

$$Monthly\ rate = \frac{Annual\ rate}{12} = \frac{0.117}{12} = 0.00975 = 0.975\ \%$$

Thus, the total interest H2M will owe at the end of one month is 0.975 % of the amount it borrowed:

$$Interest = Principal * Interest\ rate$$

$$Interest = \$10\ 887.61 * 0.00975 = \$106.15$$

Its net funds after paying the origination fee and creating the compensating balance are the \$10 269 it needs, so it is paying \$106.15 in interest plus the \$96 in origination in order to get a \$10 269 loan.

The one-month effective interest rate of the loan is:

$$Interest\ rate\ per\ period = \frac{Interest + Origination\ fee}{Discounted\ payment}$$

$$Interest\ rate\ per\ period = \frac{\$106.15 + \$96}{\$10\ 269} = 1.969\ \%$$

The effective annual interest rate is:

$$EAR = (1 + 0.01969)^{12} - 1 = 26.36\ \%$$

Alternative C

Again, H2M will need to borrow more than \$10 269 in order to pay the origination fee. With a 1.2 % origination fee, it will need to borrow an amount that, after subtracting the origination fee, is large enough to pay the discounted payment:

$$Amount\ to\ borrow - Origination\ fee\ = Discounted\ payment$$

$$Amount\ to\ borrow - \overbrace{(r * Amount\ to\ borrow)}^{\text{Origination fee}} = Discounted\ payment$$

$$Amount\ to\ borrow * (1 - r)\ = Discounted\ payment$$

$$Amount\ to\ borrow = \frac{Discounted\ payment}{(1 - r)}$$

$$Amount\ to\ borrow = \frac{\$10\ 269}{(1 - 0.012)} = \$10\ 393.72$$

In order to have \$10 269 available after paying the origination fee H2M will need to borrow \$10 393.72.

The bank is charging 15.3 % APR, compounding monthly, which translates into a monthly rate of

$$Monthly\ rate = \frac{Annual\ rate}{12} = \frac{0.153}{12} = 0.01275 = 1.275\ \%$$

So, at the end of the month H2M will owe in interest

$$\$10\ 393.72 * 0.01275 = \$132.52$$

Its net funds after paying the origination fee are the \$10 269 it needs, so it is paying \$132.52 in interest plus an origination fee of 1.2 % (\$10 393.72 * 0.012 = \$124.72) in order to get a \$10 269 loan.

The one-month effective interest rate therefore is:

$$Interest\ rate\ per\ period = \frac{Interest + Origination\ fee}{Discounted\ payment}$$

$$Interest\ rate\ per\ period = \frac{\$132.52 + \$124.72}{\$10\ 269} = 2.505\ \%$$

The effective annual interest rate is:

$$EAR = (1 + 0.02505)^{12} - 1 = 34.57\ \%$$

Now we can summarize each alternative's effective annual interest rate:

Alternative	EAR
A	30.59 %
B	26.36 %
C	34.57 %

Thus, alternative B, with the lowest effective annual rate, is the best option for H2M.

Exercise 27.6

Hand-to-Mouth (H2M) is currently cash-constrained and must make a decision about whether to delay paying one of its suppliers or take out a loan. They owe the supplier $10 000 with terms of 2/10 Net 40, so the suppliers will give them a 2 % discount if they pay by today (when the discounts period expires after 10 days). Alternatively, they can pay the full $10 000 in one month when the invoice is due. H2M is considering three options:

Alternative A: Forgo the discount on its trade credit agreement, wait and pay the full $10 000 in one month.

Alterative B: Borrow the money needed to pay its supplier today from Bank A, which has offered a one-month loan at an APR of 12 %. The bank will require a (no-interest) compensating balance of 5 % **of the face value of the loan** and will charge a $100 loan origination fee. Because H2M has no cash, it will need to borrow the funds to cover these additional amounts as well.

Alternative C: Borrow the money needed to pay its suppliers today from Bank B, which has offered a one-month loan at an APR of 15 %. The loan has an 1 % origination fee, which again H2M need to borrow to cover.

Which alternative is the cheapest source of financing for Hand-to-Mouth?

Alternative A

The interest rate per period is calculated as follows:

$$Interest\ rate\ per\ period = \frac{Discount}{Discounted\ payment}$$

$$Interest\ rate\ per\ period = \frac{0.02 * \$10\ 000}{\$10\ 000 - (0.02 * \$10\ 00)}$$

$$= \frac{\$200}{\$10\ 000 - \$200} = 2.0408\ \%$$

The loan period is one month, so the effective annual cost is:

$$EAR = (1 + Interest\ rate\ per\ period)^{number\ of\ periods\ in\ one\ year} - 1$$

$$EAR = (1 + 0.020408)^{12} - 1 = 26.89\ \%$$

Alternative B

Hand-to-Mouth will need to borrow $9 800 to pay its supplier today, as well as the $100 loan origination fee. The total borrowing amount is therefore:

$$\$9\ 800 + \$100 = \$9\ 900$$

In addition, H2M will need to meet its compensating balance requirement, which is 5 % of the borrowing amount. So, in order to have $9 900 available to spend (the $9 800 it needs to pay supplier plus the $100 origination fee), it will have to borrow an amount that, after subtracting the compensating balance requirements, is large enough to pay the discounted payment and the origination fee:

$$Amount\ to\ borrow\ - Compensating\ balance\ requirements$$
$$= Discounted\ payment + Origination\ fee$$

$$Amount\ to\ borrow\ - \overbrace{(r * Amount\ to\ borrow)}^{\text{Compensating balance requirements}}$$
$$= Discounted\ payment + Origination\ fee$$

$$Amount\ to\ borrow\ (1 - r) = Discounted\ payment + Origination\ fee$$

$$Amount\ to\ borrow = \frac{Discounted\ payment + Origination\ fee}{(1 - r)}$$

$$Amount\ to\ borrow = \frac{\$9\ 800 + \$100}{(1 - 0.05)} = \$10\ 421.05$$

This will allow H2M to set aside 5 % of the loan (\$10 421.05 * 0.05 = \$521.05) and still have \$10 421.05 - \$521.05 = \$9 900 to pay the origination fee and have a net loan of \$9 800.

The bank is charging 12 % APR, compounded monthly, which translates into a monthly rate of:

$$Monthly\ rate = \frac{Annual\ rate}{12} = \frac{0.12}{12} = 0.01 = 1\ \%$$

Thus, the total interest H2M will owe at the end of one month is 1 % of the amount it borrowed:

$$Interest = Principal * Interest\ rate$$
$$Interest = \$10\ 421.05 * 0.01 = \$104.21$$

Its net funds after paying the origination fee and creating the compensating balance are the \$9 800 it needs, so it is paying \$104.21 in interest plus the \$100 in origination fees in order to get a \$9 800 loan.

The one-month effective interest rate of the loan is:

$$Interest\ rate\ per\ period = \frac{Interest + Origination\ fee}{Discounted\ payment}$$

$$Interest\ rate\ per\ period = \frac{\$104.21 + \$100}{\$9\ 800} = 2.08\ \%$$

The effective annual interest rate is:

$$EAR = (1 + 0.0208)^{12} - 1 = 28.02\ \%$$

Alternative C

Again, H2M will need to borrow more than \$9 800 in order to pay the origination fee. With a 1 % origination fee, it will need to borrow an amount that, after subtracting the origination fee, is large enough to pay the discounted payment:

$$Amount\ to\ borrow - Origination\ fee\ = Discounted\ payment$$

$$Amount\ to\ borrow - \overbrace{(r * Amount\ to\ borrow)}^{\text{Origination fee}} = Discounted\ payment$$

$$Amount\ to\ borrow * (1 - r)\ = Discounted\ payment$$

$$Amount\ to\ borrow = \frac{Discounted\ payment}{(1 - r)}$$

$$Amount\ to\ borrow = \frac{\$9\ 800}{(1 - 0.01)} = \$10\ 888.89$$

In order to have \$9 800 available after paying the origination fee H2M will need to borrow \$10 888.89.

The bank is charging 15 % APR, compounding monthly, which translates into a monthly rate of

$$Monthly\ rate = \frac{Annual\ rate}{12} = \frac{0.15}{12} = 0.0125 = 1.25\ \%$$

So, at the end of the month H2M will owe in interest

$$\$10\ 888.89 * 0.0125 = \$136.11$$

Its net funds after paying the origination fee are the $9 800 it needs, so it is paying $136.11 in interest plus an origination fee of 1 % ($10 888.89 * 0.01 = $108.89) in order to get a $9 800 loan.

The one-month effective interest rate therefore is:

$$Interest\ rate\ per\ period = \frac{Interest + Origination\ fee}{Discounted\ payment}$$

$$Interest\ rate\ per\ period = \frac{\$136.11 + \$108.89}{\$9\ 800} = 2.50\ \%$$

The effective annual interest rate is:

$$EAR = (1 + 0.025)^{12} - 1 = 34.49\ \%$$

Now we can summarize each alternative's effective annual interest rate:

Alternative	EAR
A	26.89 %
B	28.02 %
C	34.49 %

Thus, alternative A, with the lowest effective annual rate, is the best option for H2M.